Technical paper

Vibrational energy loss analysis in battery tab ultrasonic welding

Bongsu Kang\textsuperscript{a,*}, Wayne Cai\textsuperscript{b}, Chin-An Tan\textsuperscript{c}

\textsuperscript{a} Indiana University – Purdue University Fort Wayne, United States
\textsuperscript{b} GM Global R&D Center, United States
\textsuperscript{c} Wayne State University, United States

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A B S T R A C T

In ultrasonic metal welding processes, high-frequency ultrasonic energy is used to generate friction and heat at the interface between weld parts to produce solid-state bonds. It has been observed that sufficient energy is required to produce proper bonding, while excessive energy can cause such quality issues as weld fracture and perforation. Therefore, it is important to have a product/process design in ultrasonic welding to ensure efficient energy conversion from ultrasonics to welding energy, minimizing energy loss in the process. In this work, vibrational energy loss associated with the longitudinal and flexural vibrations of the Cu coupon during ultrasonic welding is studied by applying one-dimensional continuous vibration models. To facilitate our modeling, experimental results from the free response of Cu coupon were obtained to determine the damping characteristics of the Cu coupon in the welding process. Our analysis shows that substantial energy loss can occur during welding due to the flexural vibration of the Cu coupon, especially when the overhang (the upper part of the Cu coupon extended from the anvil) of the Cu coupon resonates at or close to the welding frequency (about 20 kHz), degrading the weld quality of battery tabs. This study contributes to understanding the fundamental dynamics of the Cu coupon during ultrasonic welding and its impact on weld quality.

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1. Introduction

Automotive battery packs for electric vehicles typically consist of hundreds of battery cells in order to meet the desired power and capacity requirements. These cells must be connected together with robust mechanical joints before being assembled into a battery pack. Joining of battery cells and battery tabs presents challenges due to the need to weld multiple, highly conductive, and dissimilar materials, with varying thickness combinations. Characteristics of various joining technologies used in the battery pack industry, such as resistance welding, laser welding, ultrasonic welding, and mechanical joining, are well summarized by Lee et al. [1]. Ultrasonic metal welding (USMW) is currently the most widely used joining technique for battery pack assembly due to its ability to join dissimilar metals, such as aluminum to copper, in an automated process at relatively low cost. Moreover, in contrast to traditional fusion welding processes, USMW is a solid-state joining process [2], without using any filler material, thus eliminating consumable materials costs and wastes and post-assembly cleaning.

In ultrasonic metal welding processes, high-frequency (e.g., 20 kHz) ultrasonic energy is used to generate oscillating shears at the interface between a sonotrode (horn) and metal sheets to produce solid-state bonds between the sheets clamped under pressure in a short period of time (less than a second). The amplitude of the oscillation is normally in the range of 5–30 μm. Physical principles of USMW are discussed by Rozenberg and Mitskevich [3]. Experimental studies of the USMW mechanisms and the resulting material microstructures can be found in the works of Devine [4], Flood [5], and Hetrick et al. [6], and numerical studies of the USMW process using FEA models are presented by, for example, Viswanath et al. [7], Siddiq and Ghassemieh [8], Elangovan et al. [9], and Lee et al. [10].

Fig. 1 shows a schematic of an ultrasonic welding unit and tooling setup for a battery pack. The weld unit consists of multiple lithium-ion battery cell pouches, each has two electrode extensions (battery tabs) sealed at the upper part of the pouch, and a bus-bar. Thin copper or aluminum sheets are used for those battery tabs. Once the battery tabs and bus-bar are aligned and sandwiched under a clamping force between the sonotrode and the anvil, electrical currents passing through the piezo–stacks cause the stacks to expand and contract (oscillate) at ultrasonic frequency. This oscillation is amplified through a booster to excite the sonotrode at a desired frequency. The amplitude of the...
sonotrode oscillation is generally controlled to be constant during welding. Basic principles of power ultrasonics can be found in the paper by [11]. Ultrasonic metal welding for battery tabs must be performed with 100% reliability in battery pack manufacturing as the failure of one weld essentially results in a battery that is inoperative or cannot deliver the required power due to the electrical short caused by the failed weld. Moreover, this stringent weld quality standard is significant for battery pack manufacturers as automotive batteries are exposed to harsh driving environment such as vibration, severe temperature, and possibly crash, all of which can affect battery performance and safety. Therefore, one of the main issues arising in ultrasonic welding of battery tabs is to ensure consistent weld quality that meets design specifications such as the electrical conductivity and shear strength of the weld [12].

There are three quality indices in USMW; i.e., bonding effectiveness between weld parts [13], cracks/perforation on the weld surfaces [13], and bulging/distortion of the weld parts [14,15]. The weld quality depends on a number of controllable factors such as mechanical and metallurgical properties of the weld parts, weld part geometry and dimensions, weld configuration, weld tool (e.g., the sonotrode and

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**Nomenclature**

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A$</td>
<td>cross-sectional area of overhang ($A = dh$)</td>
</tr>
<tr>
<td>$a$</td>
<td>sonotrode amplitude</td>
</tr>
<tr>
<td>$a_l$</td>
<td>displacement of anvil in the longitudinal direction of overhang</td>
</tr>
<tr>
<td>$a_f$</td>
<td>displacement of anvil in the transverse direction of overhang</td>
</tr>
<tr>
<td>$c$</td>
<td>viscous damping coefficient</td>
</tr>
<tr>
<td>$c_0$</td>
<td>phase velocity ($= \sqrt{E/\rho}$)</td>
</tr>
<tr>
<td>$d$</td>
<td>width of overhang</td>
</tr>
<tr>
<td>$E$</td>
<td>Young’s modulus (110 GPa for copper)</td>
</tr>
<tr>
<td>$h$</td>
<td>thickness of overhang</td>
</tr>
<tr>
<td>$I$</td>
<td>second area moment of inertia of overhang ($I = dh^3/12$)</td>
</tr>
<tr>
<td>$F_0$</td>
<td>inertial body force in the transverse direction of overhang (kN/m) (see Fig. 6)</td>
</tr>
<tr>
<td>$P_0$</td>
<td>inertial body force in the longitudinal direction of overhang (kN/m) (see Fig. 5)</td>
</tr>
<tr>
<td>$L$</td>
<td>overhang length (see Fig. 7)</td>
</tr>
<tr>
<td>$u$</td>
<td>longitudinal displacement of overhang</td>
</tr>
<tr>
<td>$U_n$</td>
<td>longitudinal normal mode function (see Eq. (20))</td>
</tr>
<tr>
<td>$w$</td>
<td>transverse displacement of overhang</td>
</tr>
<tr>
<td>$W$</td>
<td>work done by excitation force</td>
</tr>
<tr>
<td>$W_d$</td>
<td>energy dissipation due to material damping</td>
</tr>
<tr>
<td>$W_n$</td>
<td>flexural normal mode function (see Eq. (45))</td>
</tr>
<tr>
<td>$x$</td>
<td>longitudinal coordinate (Figs. 5 and 6)</td>
</tr>
<tr>
<td>$\gamma_n$</td>
<td>wavenumber</td>
</tr>
<tr>
<td>$\rho$</td>
<td>mass density (8940 kg/m$^3$ for copper)</td>
</tr>
<tr>
<td>$\Omega$</td>
<td>sonotrode frequency (rad/s) ($\Omega = 2\pi f_s = \text{sonotrode frequency (Hz)}$)</td>
</tr>
<tr>
<td>$\omega_n$</td>
<td>natural frequency of the $n$th mode (rad/s) (see Eq. (19) and (44))</td>
</tr>
<tr>
<td>$\zeta_n$</td>
<td>damping ratio for the $n$th mode (see Eqs. (27) and (51))</td>
</tr>
</tbody>
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**Fig. 1.** Schematic of the weld unit and ultrasonic welding setup.
anvil) design, and welding process parameters – sonotrode amplitude, clamping pressure, and welding energy. In this regard, a significant amount of research has been made; for example, Lee et al. [13,16], Li et al. [17], and Zhao et al. [18].

The weld quality also depends on a number of uncontrollable factors such as weld tool alignment, tool wear, work part surface variations and contaminations, and, as a unique characteristics in ultrasonic welding, the dynamics of the ultrasonic welding system, particularly the structural vibrations of weld parts and supporting structures (tools and fixtures). By experimental and finite element analyses, Jagota and Dawson [19] showed that the bonding strength of thin-walled thermoplastic parts by ultrasonic welding is strongly influenced by the lateral vibration of the weld parts. The impact of waveform designs, by controlling the wavelength of the ultrasonic input, on vibration response reduction of weld parts for the battery welding system was studied by Lee et al. [16]. The effect of dynamic responses of weld parts on the sonotrode force required for welding in USMW was studied by Kang et al. [14]. In our prior work [20], extensive experiments were also conducted to gain understanding of the vibration characteristics of the ultrasonic welding process of battery tabs. The experiments were designed to weld three tabs (which were not attached to battery cell pouches; the tab ends were clamped to a fixture) to the bus-bar (Cu coupon). Vibration response of the welder system was measured by a high precision (resolution of 2 nm) Polytec single-point laser vibrometer. The vibrometer was synchronized with the welder system and a National Instruments DAQ system. The frequency of the sonotrode oscillation during welding was about 20 kHz and data were sampled at 200 kHz. Data were post-processed using the laser vibrometer software and MATLAB to obtain the response, FFT and other critical characteristics. In a set of experiments, the stiffness of the anvil was deliberately reduced, and Polytec laser vibrometer measured several microns more anvil vibration (almost doubled), adversely affecting the weld quality. In particular, some of the weld spots could debond and the tabs could also bulge. The above study demonstrated that undesirable anvil vibrations, even at the level of a few microns, could have significant adverse effects on the weld quality. The above experiment also clearly indicated that although the sole vibration source is from the sonotrode, anvil does vibrate due to the coupling effect since the sonotrode applies high clamping force onto the work parts against the anvil.

Since the more system vibrates, the more energy loss is. Thus, the objective of this work is to study the observed phenomena from vibrational energy point of view. We believe it is very important to minimize vibrational energy loss by carefully designing the weld parts and tools, especially to avoid resonance of any of the weld parts and tools as considerable amounts of energy are consumed through resonant vibrations. Therefore, the present work examines the vibration energy loss of the bus-bar coupon due to the longitudinal and flexural vibrations of the overhang (upper part of the bus-bar extended from the anvil) during ultrasonic welding and assess the effects of this energy loss on the weld strength of battery tabs. The overhang of the bus-bar is modeled as a thin beam extended in the direction parallel to the excitation direction of the sonotrode. Section 2 presents the principle of work and energy loss for forced longitudinal and flexural vibrations of thin beams. In Section 3, vibrational energy loss associated with the vibration of the overhang is discussed. Summary and conclusions are presented in Section 4.

2. Theory

In ultrasonic welding of battery tabs, the thickness of the bus-bar coupon is much smaller than its other dimensions and the loading conditions are symmetric. Thus, in the present study, the overhang of the bus-bar coupon is modeled as a thin, internally damped beam extended parallel to the direction of the excitation of the sonotrode. An overview of the theory needed to calculate the vibrational energy loss of the bus-bar coupon is presented in this section. Using the modal analysis approach, the energy loss is expressed as a sum of the modal energies. Hence, in order to understand the frequency characteristics of modal energy loss, it is essential to first review the basic results for the single degree-of-freedom model.

2.1. Work-energy relations for damped harmonic motions

Consider a single degree-of-freedom (SDOF) damped oscillatory system subjected to a harmonic force $F(t)$ as shown in Fig. 2, where $x$ denotes the displacement, $m$ the mass, $k$ the stiffness, and $c$ the viscous damping coefficient (or damping constant). The governing equation of motion for the mass is

$$\ddot{x} + 2\xi \omega_n \dot{x} + \omega_n^2 x = \frac{F_0}{m} \sin \Omega t$$

where $\omega_n = \sqrt{k/m}$ denotes the natural frequency of the system and $\xi = c/2m\omega_n$ is the damping ratio (or damping factor).

The response of the system to the harmonic excitation can be written as

$$x(t) = X_0 e^{-\omega_d t} \sin(\omega_d t + \phi_d) + X \sin(\Omega t - \phi)$$

where the first term is the free response, with $X_0$ and $\phi_d$ determined from the initial conditions (see Eq. (A3) in Appendix A), and

$$\omega_d = \omega_n \sqrt{1 - r^2} \quad \phi = \tan^{-1} \left( \frac{2\Omega r}{\sqrt{1 - r^2}} \right)$$

$$r = \frac{\Omega}{\omega_n}$$

Fig. 2. Single-DOF damped oscillatory system subjected to a harmonic force.
\( \omega_d \) is the damped natural frequency. The mechanical work performed by the harmonic force \( F(t) \) on the system over the first \( N \) loading cycles can be written as

\[
W = \int_0^{2N\pi} F(t)\dot{x}(t) \, dt
= \int_0^{2N\pi} 2N\pi e^{-\zeta\omega nt} \sin \omega t \cos(\omega t + \phi) - \zeta \omega n t \sin(\omega t + \phi) \, dt + F_0 \Omega \int_0^{2N\pi} \sin \Omega t \cos(\Omega t - \phi) \, dt
= \left( \text{work done on the free response up to } t = \frac{2N\pi}{\zeta} \right) + \pi NF_0X \sin \phi \tag{4}
\]

It can easily be seen from Eq. (4) that, for sufficiently large number of cycles \( N \), the work done on the free response vanishes due to the presence of the decay term \( e^{-\zeta\omega nt} \), and the mass vibrates in steady-state. Note also that one can determine \( W \) from direct integration of Eq. (1) as follows

\[
W = \int_0^{t_s} F(t) \dot{x}(t) \, dt = \left( \frac{1}{2} \frac{mx^2}{\zeta} + \frac{1}{2} \frac{kx^2}{\zeta} \right)_{t = \frac{2N\pi}{\zeta}}^{t_s} + \int_0^{2N\pi} \frac{dx^2}{dt} \, dt \tag{5}
\]

The first two terms in Eq. (5) are the kinetic \( (T) \) and elastic \( (V) \) energy of the system, respectively, and the last term is the dissipated energy \( (W_d) \) by viscous damping. Hence, the work done by the excitation force is dispensed into the kinetic and elastic energies and dissipation. Fig. 3 plots the breakdown of the non-dimensional work \( W \) in terms of \( T, V, \) and \( W_d \) up to the settling time \( t_s \) as a function of the frequency ratio \( r \) for two different values of damping ratio \( \zeta \). Note that \( t_s \) is the time required for the system to reach its steady state and approximated as \( t_s = \frac{4}{\zeta \omega n} \) for an underdamped \( (\zeta < 1) \) system. Note also that the graphs in Fig. 3 are plotted on different scales. It can be seen that \( W_d \) contributes to most of \( W \), particularly for \( r > 1 \), meaning that most of work done is dissipated by damping. In other words, unless the system is excited at a frequency much lower than its natural frequency, a relatively small amount of energy from the total work done by the harmonic force is consumed to build up the kinetic or elastic energy of the system, and that these can be neglected for the steady-state vibrational energy loss analysis as discussed below. It is also important to realize that, for a lightly damped system (such as \( \zeta = 0.1 \)) as shown in Fig. 3(a), the maximum \( W_d \) occurs at \( r \approx 1 \) and this magnitude is much larger than the one for a system with a larger damping ratio (see Fig. 3(b)).

From Eq. (2), for a system or a modal coordinate with large damping ratio or natural frequency, the steady-state response \( x(t) = X \sin(\Omega t - \phi) \) is quickly attained. The mechanical work performed by the excitation \( F(t) \) per cycle in steady-state vibration is given by the second term in Eq. (4)

\[
W_{ss} = \int_0^{2\pi/\Omega} F(t)\dot{x}(t) \, dt = \pi F_0X \sin \phi \tag{6}
\]

It can be seen from the above expression that \( W_{ss} = 0 \) when \( \phi = 0 \), implying that the work performed by a harmonic force in phase with a harmonic displacement is zero after a whole cycle. In addition, note that \( W_{ss} \) is maximum when \( \phi = 90^\circ \), and that this occurs only

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**Fig. 3.** Work performed by \( F(t) \) is distributed into kinetic energy, elastic energy, and energy dissipated by damping.
at resonance where the harmonic force and velocity are completely in phase. The energy dissipated by damping per steady-state cycle can be found in terms of work done by the damping force as follows:

\[
W_d = \int_{\phi}^{\phi + 2\pi} c\dot{X}^2(t) dt = c\Omega^2 X^2 \int_{\phi}^{\phi + 2\pi} \cos^2(\Omega t - \phi) dt = \pi c\Omega X^2
\]  

(7)

When substituting \( X, \phi, \) and \( r \) in Eq. (3) into Eqs. (6) and (7), it can be shown that

\[
W_{st} = W_d = \frac{F_0^2 c}{K (1 - r^2)^2 + (2\xi r)^2}
\]

(8)

Shown in Fig. 4 is the energy dissipated by damping per steady-state cycle for different values of the damping ratio \( \xi \). It can be clearly seen that a large amount of energy can be dissipated by damping at resonance even when the system or the modal coordinate is lightly damped. Note that the amount of energy dissipated by damping decreases with increasing damping at resonance.

2.2. Energy dissipation by material damping in longitudinal vibration

Consider a thin, infinitely long, undamped, straight bar with a uniform cross-section subjected to an arbitrarily distributed axial body force \( P(x, t) \) (measured as a force per unit length) as shown in Fig. 5(a). The equation governing the longitudinal vibration of the bar can be found as [21],

\[
\rho A \frac{\partial^2 u}{\partial t^2} = EA \frac{\partial^2 u}{\partial x^2} + P(x, t)
\]

(9)

where \( u = u(x, t) \) denotes the axial displacement of a cross-section, \( x \) the spatial coordinate, \( t \) the time, \( E \) the Young’s modulus, \( A \) the cross-sectional area, and \( \rho \) the mass density of the bar. In the absence of the body force, Eq. (9) reduces to the classical wave equation:

\[
\frac{1}{c_0^2} \frac{\partial^2 u}{\partial t^2} = \frac{\partial^2 u}{\partial x^2}, \quad c_0 = \sqrt{\frac{E}{\rho}}
\]

(10)

where \( c_0 \) is the phase velocity (or bar velocity) at which longitudinal waves propagate. Typical phase velocities in most metals are quite high compared to the velocity of sound in air of 340 m/s, for example copper has \( c_0 = 3508 \) m/s.

The general solution of Eq. (9) can be found by assuming that

\[
u(x, t) = U(x)G(t)
\]

(11)

which leads to

\[
U(x) = C_1 \cos \gamma x + C_2 \sin \gamma x
\]

(12)

\[
G(t) = D_1 \cos \omega t + D_2 \sin \omega t
\]

(13)
where the radial frequency $\omega$, wavenumber $\gamma$, and wavelength $\lambda$ (the distance between two successive points of constant phase) are related by

$$\omega = \xi_0 \gamma = 2\pi \frac{\xi_0}{\lambda}$$

(14)

The arbitrary constants in Eqs. (12) and (13) depend on the boundary conditions and initial conditions. For example, consider a bar fixed at one end ($x = 0$) and free at the other end ($x = L$). The fixed boundary condition at $x = 0$ implies that the displacement at the end must be zero, therefore

$$u(0, t) = U(0)G(t) = C_1 G(t) = 0$$

(15)

Since $G(t) \neq 0$ for all time, Eq. (16) dictates that $C_1 = 0$. In addition, the free boundary condition at $x = L$ requires that the stress at the end must vanish; i.e.,

$$EA \frac{d^2 u(L, t)}{dx^2} = EA \frac{d U(L)}{dx} + \gamma G(t) = 0$$

(16)

Since $C_2 \neq 0$, $\gamma \neq 0$, and $G(t) \neq 0$, it can be found that

$$\cos \gamma L = 0$$

(17)

which is the frequency equation for the fixed-free bar under longitudinal vibration. Eq. (17) is satisfied only when

$$\gamma_n = \frac{(2n - 1)\pi}{2L} \quad n = 1, 2, 3, \ldots$$

(18)

Thus, the natural frequencies of the system are

$$\omega_n = \frac{(2n - 1)\pi \xi_0}{2L} \quad n = 1, 2, 3, \ldots$$

(19)

These represent the discrete frequencies at which the system is capable of undergoing resonance. For a given $n$, the vibrational pattern (called the $n$th normal mode or mode shape) of the bar is described by

$$U_n(x) = \sin \gamma_n x \quad n = 1, 2, 3, \ldots$$

(20)

Combining the time and spatial dependence for a given $n$, the assumed solution in Eq. (11) becomes

$$u_n(x, t) = (D_{1n} \cos \omega_n t + D_{2n} \sin \omega_n t) \sin \gamma_n x$$

(21)

The general solution is then obtained by superposing all particular solutions as

$$u(x, t) = \sum_{n=1}^{N} u_n(x, t) = \sum_{n=1}^{N} (D_{1n} \cos \omega_n t + D_{2n} \sin \omega_n t) \sin \gamma_n x$$

(22)

where the coefficients $D_{1n}$ and $D_{2n}$ are to be determined by applying the initial conditions of the bar.

Now, consider a fixed-free bar of length $L$ (see Fig. 5(b)) with Kelvin–Voigt material damping (internal damping) and subjected to an axially distributed force $P(x, t)$. The corresponding equation of motion governing the longitudinal vibration takes the following form

$$\rho A \frac{\partial^2 u}{\partial t^2} = c \frac{\partial^2 u}{\partial x^2} + EA \frac{\partial^2 u}{\partial x^2} + P(x, t)$$

(23)

Note that Kelvin–Voigt damping is a viscoelastic model commonly used for metals with relatively small damping. In this model, the stress $\sigma$, strain $\varepsilon$, and its rate of change with respect to time are related by:

$$\sigma = E\varepsilon + c \frac{\partial \varepsilon}{\partial t}$$

(24)

where $c$ represents the viscosity of the material. The solution to Eq. (23) can be written in terms of the normal modes associated with undamped system as

$$u(x, t) = \sum_{n=1}^{N} q_n(t) U_n(x)$$

(25)

where $U_n(x)$ is the normal mode function for the fixed-free bar as shown in Eq. (20) and $q_n(t)$, referred to as time-dependent generalized coordinates or modal coordinates [22] in modal analysis of discrete systems, satisfies the following equation:

$$\ddot{q}_n + 2\xi_n \omega_n \dot{q}_n + \omega_n^2 q_n = Q_n(t)$$

(26)

$$Q_n(t) = \frac{1}{\rho A \omega_n} \int_0^L P(x, t) U_n(x) dx$$

$$\xi_n = \frac{\omega_n}{2EA} \quad \alpha_n = \int_0^L U_n^2(x) dx$$

(27)

Note that the basic form of Eq. (26) is the vibration of a single DOF represented by Eq. (1). $Q_n(t)$ is the generalized force or modal force in modal analysis of discrete systems associated with $q_n(t)$ and $\xi_n$ is the modal damping ratio. If $P(x, t) = P_0 \sin \Omega t$, i.e., a uniformly distributed harmonic body force, one can find that

$$q_n(t) = \frac{P_0}{\rho A \omega_n \alpha_n} \int_0^t U_n(x) dx \int_0^t \sin \Omega \tau e^{-\xi_n \tau} \sin \omega_n \sqrt{1 - \xi_n^2} (t - \tau) d\tau$$

(28)
Once \( u(x, t) \) in Eq. (25) is found, the mechanical work performed by the axial body force on the harmonic motion of the thin bar over a time span of \( t_0 \) can be determined in a straight manner as follows

\[
W = \int_0^L \int_0^{t_0} P(x, \tau) \dot{u}(x, \tau) \, d\tau \, dx = \int_0^L \int_0^{t_0} P(x, \tau) \left( \sum_{n=1}^{N} \dot{q}_n(\tau) U_n(x) \right) \, dx \, d\tau = \rho A \sum_{n=1}^{\infty} \alpha_n \int_0^{t_0} \dot{q}_n(\tau) Q_n(\tau) \, d\tau
\]  

(29)

In addition, the kinetic energy at \( t = t_0 \) is

\[
T = \frac{1}{2} \rho A \int_0^L \left( \frac{\partial u}{\partial t} \right)^2 \, dx = \frac{1}{2} \rho A \sum_{n=1}^{N} \alpha_n \dot{q}_n^2(t_0)
\]

(30)

and the elastic energy at \( t = t_0 \) is

\[
V = \frac{1}{2} EA \int_0^L \left( \frac{\partial u}{\partial x} \right)^2 \, dx = \frac{1}{2} EA \sum_{n=1}^{N} \alpha_n \dot{q}_n^2(t_0) \int_0^L U_n^2(x) \, dx
\]

(31)

Lastly, the energy dissipated by material damping over \( t_0 \) can be found as

\[
W_d = -\int_0^L \int_0^{t_0} c \left( \frac{\partial^2 u}{\partial x^2} \right) \left( \frac{\partial u}{\partial t} \right) \, dx \, d\tau = -c \int_0^L \int_0^{t_0} \left( \sum_{n=1}^{N} \dot{q}_n(\tau) U_n(x) \right) \left( \sum_{n=1}^{N} \dot{q}_n(\tau) U_n(x) \right) \, dx \, d\tau = 2 \rho A \sum_{n=1}^{N} \alpha_n \xi_n \omega_n \int_0^{t_0} \dot{q}_n^2(\tau) \, d\tau
\]

(32)

It should be noted that the work done \( W \) by the harmonic body force \( P(x, t) \) in Eq. (29) and the energy dissipated by damping \( W_d \) in Eq. (32) include both the transient and steady-state responses as the solution in Eq. (25) includes both response solutions. Moreover, energies in Eqs. (29)–(32) are expressed in terms of the modal energies, i.e., energies associated with the modal coordinates, whose fundamental characteristics in the frequency domain have been described in Section 2.1.

### 2.3. Energy dissipation by material damping in flexural vibration

Consider a thin bar, clamped at \( x = 0 \) and free at \( x = L \), under flexural (transverse) vibration due to a body force \( F(x, t) \) distributed over the span, as shown in Fig. 6. Denoting \( x \) as the spatial variable and \( t \) the time variable, the equation governing the transverse displacement \( \tilde{w}(x, t) \) under the flexural vibration of the bar is [21]

\[
\rho A \frac{\partial^2 \tilde{w}}{\partial t^2} + EI \frac{\partial^4 \tilde{w}}{\partial x^4} = F(x, t)
\]

(33)

where \( I \) is the second area moment of inertia of the bar; i.e., \( I = dh^3/12 \). In free vibration, the general solution of Eq. (33) can be found by assuming that

\[
\tilde{w}(x, t) = W(x) G(t)
\]

(34)

Application of the above-assumed solution to Eq. (33) leads to

\[
W(x) = C_1 \cos \gamma x + C_2 \cosh \gamma x + C_3 \sin \gamma x + C_4 \sinh \gamma x
\]

(35)

\[
G(t) = D_1 \cos \omega t + D_2 \sin \omega t
\]

(36)

where the radial frequency \( \omega \) and wavenumber \( \gamma \) are related by

\[
\omega^2 = \frac{EI}{\rho A} \gamma^4
\]

(37)

The arbitrary constants in Eqs. (35) and (36) depend on the boundary conditions and initial conditions. For clamped boundary conditions at \( x = 0 \), the displacement and slope at the end must be zero. Hence,

\[
\tilde{w}(0, t) = W(0) G(t) = (C_1 + C_2) G(t) = 0
\]

(38)

\[
\frac{\partial \tilde{w}(0, t)}{\partial x} = \tilde{w}'(0) G(t) = \gamma (C_3 + C_4) G(t) = 0
\]

(39)
In addition, the free boundary conditions at \( x = L \) require that the bending moment and shear force at the end must vanish; i.e.,

\[
EI \frac{d^2w(x, t)}{dx^2} = EW''(x, t)G(t) = -E\bar{\gamma}\partial^2C_1 \cos \gamma L - C_2 \cosh \gamma L + C_3 \sin \gamma L - C_4 \sinh \gamma L)(1) = 0
\]  
\[
EI \frac{d^2w(x, t)}{dx^2} = EW''(x, t)G(t) = E\bar{\gamma}\partial^3C_1 \sin \gamma L + C_2 \sinh \gamma L - C_4 \cos \gamma L + C_4 \cosh \gamma L)(1) = 0
\]  
\[
(40)
\]  
\[
(41)
\]

Since \( \gamma \neq 0 \) and \( G(t) \neq 0 \) \( \forall t \), in order for Eqs. (38)–(41) to have a nontrivial solution, wavenumber \( \gamma \) must satisfy the following equation

\[
\cos \gamma L + 1 = 0
\]

which is the frequency equation for the clamped-free thin bar under flexural vibration. The lowest five values of \( \gamma L \) satisfying Eq. (42) are

\[
\gamma L = \{ 1.8751 4.6941 7.8548 10.9955 14.1372 \} \quad n = 1, 2, \ldots, 5
\]

For \( n > 5 \), \( \gamma L \approx (2n - 1)\pi/2 \). From Eq. (37), the natural frequency of the clamped-free thin bar under flexural vibration are thus

\[
\omega_n = \frac{\gamma L}{L^2} \sqrt{\frac{EI}{\rho A}} \quad n = 1, 2, 3, \ldots
\]

(43)

These represent the discrete frequencies at which the system is capable of undergoing resonance. For a given \( n \), the \( n \)th normal mode (mode shape) of the bar is

\[
W_n(x) = \cos \gamma_n x - \cos \gamma_n x - \sinh \gamma_n L - \sin \gamma_n L + \cos \gamma_n L \sinh \gamma_n x - \sin \gamma_n x \quad n = 1, 2, 3, \ldots
\]

(44)

Combining the time and spatial dependence for a given \( n \), the assumed solution in Eq. (34) becomes

\[
w_n(x, t) = (D_{n1} \cos \omega_n t + D_{n2} \sin \omega_n t)W_n(x)
\]

(45)

The general solution is then obtained by superposing all particular solutions as

\[
w(x, t) = \sum_{n=1}^{N} w_n(x, t) = \sum_{n=1}^{N} (D_{n1} \cos \omega_n t + D_{n2} \sin \omega_n t)W_n(x)
\]

(46)

where the coefficients \( D_{n1} \) and \( D_{n2} \) are to be determined by applying the initial conditions of the thin bar.

Now, consider a clamped-free bar of length \( L \) with Kelvin–Voigt material damping and subjected to an distributed body force \( F(x, t) \). The corresponding equation of motion governing the longitudinal vibration takes the following form

\[
\rho A \frac{\partial^2 w(x, t)}{\partial t^2} + c \frac{\partial^2 w(x, t)}{\partial x^2} + EI \frac{\partial^4 w(x, t)}{\partial x^4} = F(x, t)
\]

(47)

In the same as determining the solution for the longitudinal vibration problem in Section 2.2, the solution to Eq. (48) can be written in terms of the normal modes associated with undamped system as

\[
w(x, t) = \sum_{n=1}^{N} q_n(t)W_n(x)
\]

(48)

where \( W_n(x) \) is the normal mode function for the clamped-free bar as shown in Eq. (45) and the \( n \)th time-dependent generalized coordinate \( q_n(t) \) satisfies the following equations

\[
\dot{q}_n + 2\xi_n \omega_n q_n + \omega_n^2 q_n = Q_n(t)
\]

(49)

Again, the basic problem of (50) is the vibration of a single DOF represented by Eq. (1). \( Q_n(t) \) is the generalized force associated with \( q_n(t) \) and \( \xi_n \) is the modal damping ratio. If \( F(x, t) = F_0 \sin \Omega t \), i.e., a uniformly distributed harmonic body force, it can be found that

\[
q_n(t) = \frac{F_0}{\rho A \omega_n \sqrt{1 - \xi_n^2}} \int_0^t \sin \Omega t \cos (\omega_n t - \tau) d\tau
\]

(51)

Once \( w(x, t) \) in Eq. (49) is obtained in terms of the normal modes, the mechanical work performed by \( F(x, t) \) on the harmonic flexural motion of the clamped-free thin bar over a time span of \( t_0 \) can be determined in a similar manner as for the previous longitudinal vibration problem

\[
W = \int_0^{t_0} \int_0^L F(x, t)w(x, t) dx dt = \int_0^{t_0} \int_0^L F(x, t) \left( \sum_{n=1}^{N} q_n(t)W_n(x) \right) dx dt = \frac{\rho A}{2} \sum_{n=1}^{N} \alpha_n \int_0^{t_0} q_n(t)Q_n(t) dt
\]

(52)

(53)

The kinetic energy at \( t = t_0 \) becomes

\[
T = \frac{1}{2} \frac{\partial W}{\partial t} \frac{\partial W}{\partial t} = \frac{1}{2} \frac{\partial W}{\partial t} \frac{\partial W}{\partial t} = \frac{1}{2} \rho A \sum_{n=1}^{N} \alpha_n \dot{q}_n^2(t_0)
\]

(54)
and the elastic energy at \( t = t_0 \) is

\[
V = \frac{1}{2} EI \int_0^L \left( \frac{\partial^2 w}{\partial x^2} \right)^2 \, dx = \frac{1}{2} EI \sum_{n=1}^{N} q_n^2(t_0) \int_0^L W_n^2 \, dx
\]

The dissipated energy dissipated by material damping over a time span of \( t_0 \) for the clamped-free thin bar under flexural vibration is

\[
W_d = \int_0^L \int_0^{t_0} \left( \frac{\partial^2 u}{\partial x^2} \right) \left( \frac{\partial w}{\partial t} \right) \, dx \, dt = c \int_0^L \int_0^{t_0} \left( \sum_{n=1}^{N} q_n(\tau) W_n(x) \right) \left( \sum_{n=1}^{N} \omega_n W_n(x) \right) \, d\tau \, dx = 2\pi A \sum_{n=1}^{N} \omega_n \int_0^{t_0} q_n^2(\tau) \, d\tau
\]

Again, it should be noted that the work done \( W \) by the harmonic body force \( F(x, t) \) in Eq. \((53)\) and the energy dissipated by damping \( W_d \) in Eq. \((56)\) include both the transient and steady-state responses as the solution in Eq. \((49)\) is not limited to the steady-state response.

3. Vibrational energy loss of the overhang

Fig. 7 shows a schematic of the current ultrasonic welding setup for battery tabs and bus-bar (U/W-channel), where the battery tabs and bus-bar are clamped between the sonotrode tips and the anvil. As discussed in Section 1, the weld quality of battery tabs, measured by pull/peel tensile forces, perforation, and bulging, is affected by the dimensions of the bus-bar, especially by the overhang length \( L \) of the bus-bar. Longitudinal and transverse vibrations (in the \( x \) and \( y \) directions, respectively) of the anvil, of the order of several microns, can cause the overhang to vibrate in those respective directions. With different \( L \), these vibrations are suspected to cause the differences in the weld quality. In this study, the vibrational energy loss associated with the overhang is estimated and correlated to the weld strength to assess the effects of the overhang length on the weld quality. We model the overhang of the bus-bar as a thin bar vibrating in both the longitudinal and transverse directions under body force excitation due to the anvil vibrations.

The following observations and assumptions are imposed in our model:

i. The vibrations of the overhang of the bus-bar are caused primarily by the vibrations of the anvil, i.e., the anvil can be considered a moving support for the overhang of the bus-bar. For this case, the excitation force at the anvil and overhang interface can be transformed into an inertial body force uniformly distributed over the span of the overhang, especially when the excitation frequency does not resonate a high vibration mode of the overhang [23]. Under the current design of the bus-bar and welding setup, \( L \leq 10 \) mm, and for such a short span length, the dominant vibration mode of the overhang is the fundamental (first) mode for both longitudinal and flexural vibrations. For example, when \( L = 10 \) mm and \( t = 0.9 \) mm (thickness), \( \omega_1 = 5.1 \) kHz and \( \omega_2 = 32 \) kHz for the flexural vibration and \( \omega_1 = 87.7 \) kHz and \( \omega_2 = 263 \) kHz for the longitudinal vibration.

ii. The thickness of the overhang is much smaller than the other dimensions of the overhang.

iii. The longitudinal and flexural vibrations of the overhang are uncoupled. This is a reasonable assumption as the natural frequencies of the longitudinal vibration are much higher than those of the flexural vibration. In addition, vibration amplitudes of the overhang can be considered very small so that linear elasticity theory is applicable.

iv. The following values of the physical parameters are used in the numerical simulation results:

- Physical properties: \( \rho = 8940 \) kg/m\(^3\), \( E = 110 \) GPa for copper
- Geometry: \( A = dh \), \( d = 45 \) mm, \( h = 0.4 - 0.9 \) mm
- Welding parameters: \( a_1 = 10 \) \( \mu \)m, \( a_2 = 5 \) \( \mu \)m, \( \Omega = 20 \) kHz
- \( P_1 = \rho A a_2 \Omega^2 = 25.4 - 57.2 \) kN/m, \( F_0 = \rho A a_2 \Omega^2 = 12.7 - 28.6 \) kN/m

![Fig. 7. Schematic of the battery tabs welding setup.](image-url)
Modal damping ratio: \( \zeta_n = \frac{1}{2} \frac{\omega_n}{\omega_n} = 0.001, 0.003, 0.01, 0.02 \) for both longitudinal and flexural modes. A series of experiments was performed to estimate the damping ratio of the bus-bar coupon in ultrasonic welding (see Appendix A) using the logarithmic decrement method in vibration. The damping ratios chosen in the numerical simulations are based on the range of estimated values of \( \zeta \) from the experiments.

### 3.1. Longitudinal vibrational energy loss

In this section, we estimate the energy dissipated by material damping in the longitudinal vibration of the overhang. From Section 2.2, the mechanical work \( W \) performed by the axially distributed body force \( P(x,t) = P_0 \sin \Omega t \) is

\[
W = \rho A \sum_{n=1}^{N} \alpha_n \int_0^{t_0} \ddot{q}_n(t)Q_n(t) \, dt
\]

and the energy \( W_d \) dissipated by material damping is

\[
W_d = 2\rho A \sum_{n=1}^{N} \alpha_n \zeta_n \omega_n \int_0^{t} \dot{q}_n(t) \, dt
\]

where for our numerical simulations, \( N = 10 \) (the first ten normal modes), and \( t_0 = 0.1 \) s (typical welding time). Fig. 8 plots \( W_d \) as a function of overhang length \( L \) of the bus-bar for \( h = 0.9 \) mm with four different modal damping ratios. Note that \( W > W_d \), however their differences are so small that they basically overlap each other on the plots, indicating that the work required to set the overhang into longitudinal motion is negligible. Key findings of the simulation results are summarized as follows:

- The minimum length for which the first longitudinal natural frequency \( (\omega_1) \) of the overhang matches with the ultrasonic excitation frequency \( (\Omega = 20 \ kHz) \) is 43.88 mm. In this situation, there can be a large amount of vibrational energy loss. However, since \( L \leq 10 \) mm in current bus-bar designs, this resonance will not occur in practice.
- For \( L \leq 20 \) mm, \( W_d \) due to longitudinal vibration of the overhang is less than 10J within the range of modal damping ratio values. Note that \( \omega_1 = 43.8 \ kHz \) for \( L = 20 \) mm.
- In practice, for \( L \leq 10 \) mm, \( W_d \) due to longitudinal vibration of the overhang is negligibly small (less than 1J) for the range of modal damping ratio values considered.
- In general, \( W_d \) increases with increasing damping. However, \( W_d \) decreases with increasing damping near resonance, as discussed in Fig. 4 for the SDOF system.

### 3.2. Flexural vibrational energy loss

Due to undesirable vibration of the anvil in the direction normal to the welding surface, the overhang of the bus-bar is subjected to transverse excitation at the welding frequency. \(^1\) The flexural vibration of the overhang caused by this excitation is suspected to lower the weld strength of battery tabs by channeling a significant amount of energy to the vibrational energy of the overhang. In this section, the amount of energy dissipated by material damping due to the flexural vibration of the overhang is estimated. From Section 2.3, the mechanical work \( W \) performed by the distributed inertial body force \( F(x,t) = F_0 \sin \Omega t \) during the flexural vibration of the overhang is

\[
W = \rho A \sum_{n=1}^{\infty} \alpha_n \int_0^{t_0} \ddot{q}_n(t)Q_n(t) \, dt
\]

\(^1\) Sub- and super harmonics of 20kHz are occasionally observed in experiments, however their spectral power is much lower.
and the energy $W_d$ dissipated by material damping is

$$W_d = 2\rho A \sum_{n=1}^{N} \kappa_n \omega_n \int_{0}^{t_0} \dot{q}_n^2(\tau) \, d\tau$$

(60)

where $N = 10$ and $t_0 = 1$ second are used in the numerical simulations. Shown in Fig. 9 are the plots of $W_d$ for the flexural vibration of the overhang as a function of its length $L$ for four different modal damping ratios and different bus-bar thickness (from 0.4 to 0.9 mm). Again, note that $W > W_d$, but their differences are very small. Every peak in the plots of Fig. 9 corresponds to a length $L$ for which the coupon overhang will resonate at the welding frequency (20 kHz). It should be noted that simulation results presented in the work are based on linear elastic vibration theory from which resonance merely predicts the conditions under which significant energy loss could occur.

- Unlike the case of longitudinal vibration of the overhang (see Fig. 8) in which the first resonant peak occurs when $L$ is large (>40 mm) which does not happen in practice, resonance of flexural vibration can occur when $L$ is between 4 and 6 mm for the range of coupon thickness considered.
Significant vibrational energy loss (of the order of hundreds of joules) can occur when one of the flexural natural frequencies of the overhang is close to the excitation frequency $\Omega = 20$ kHz. For example, with thickness $h = 0.6$ mm: for $L = 4.1$ mm, the first natural frequency $\omega_1 = 20.225$ kHz, for $L = 10.3$ mm, the second natural frequency $\omega_2 = 20.083$ kHz, and for $L = 17.3$ mm, the third natural frequency $\omega_3 = 19.933$ kHz; and with $h = 0.9$ mm: for $L = 5.1$ mm, $\omega_1 = 19.607$ kHz; for $L = 12.6$ mm, $\omega_2 = 20.131$ kHz. Such energy loss could account for reduced peel tensile strength.

Based on the results of our analysis, the overhang of the bus-bar should be kept as short as possible to minimize vibrational energy loss due to the flexural vibration of the overhang during welding. For the coupons under study ($h = 0.9$ mm), an overhang length of 2 mm or smaller results in an insignificant amount of energy loss.

3.3. Effect of thickness of bus-bar on vibrational energy loss

From Eq. (19), since the longitudinal natural frequencies of the overhang are independent of the cross-sectional dimensions, we note that the energy dissipated by damping ($W_d$) due to the longitudinal vibration of the overhang is linearly proportional to the cross-sectional area of the overhang as shown in Eq. (58). Therefore, regardless of the cross-sectional dimensions, $W_d$ peaks at the same length. In other words, increasing the overhang thickness (or width) will simply translate the $W_d$ curves in Fig. 8 upward, without altering the shape of the curves.

However, the flexural natural frequencies of the overhang vary with the cross-sectional dimensions as shown in Eq. (44). Therefore, as illustrated in Fig. 10, the overhang length that results in a modal frequency matching with the excitation frequency (20 kHz) shifts as the overhang thickness changes. In other words, depending on the overhang thickness, $W_d$ peaks at a different overhang length. Moreover, it is seen from Eq. (60) that $W_d$ increases with the cross-sectional area $A = dh$ (i.e., $W_d$ increases with the thickness) of the overhang; however, not linearly since $\omega_n$ in Eq. (60) also depends on $A$.

4. Summary and conclusion

The vibrational energy loss of the bus-bar associated with the longitudinal and flexural vibrations of the overhang (the upper part of the bus-bar extended from the anvil) is studied in the present work by applying one-dimensional continuous vibration models. The overhang of the bus-bar is modeled as a thin bar under both the longitudinal and flexural excitations from the anvil. Experiments were also performed to obtain the damping characteristics of the coupon in order to provide realistic values of the material damping ratio in the modeling. Our results lay a foundation for a scientific understanding of the bus-bar dynamics during ultrasonic welding and its potential impact on the weld strength, thus providing guidelines for design and welding of battery tabs. Major findings are summarized as follows:

1. The energy loss due to longitudinal vibration of the bus-bar overhang is negligible.
2. A substantial amount of energy loss can occur due to the flexural vibration of the bus-bar overhang during welding when the overhang resonates at the welding frequency (about 20 kHz).
3. Vibrational energy loss through the bus bar can be significantly reduced by (a) suppressing the anvil vibration; and (b) optimizing the overhang length to avoid vibration resonance.
4. The energy loss is nil when there is no overhang.
5. The energy loss could account for the reduction of the weld strength.

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Appendix A. Estimation of damping in bus-bar coupon

Characteristics of material damping in mechanical systems can be estimated by theory of vibration. One of the most commonly employed techniques is the logarithmic decrement method [24]. Consider the free vibration of a single degree-of-freedom (SDOF) system with viscous damping, see Fig. A.1. The equation of motion is

\[ mi\ddot{x}(t) + ci + ki = 0 \]  

(A1)

with initial conditions of \( x(0) = x_0 \), \( \dot{x}(0) = v_0 \). We note that this SDOF model is fundamentally the same as those modal Eqs. (26) and (50) for the longitudinal and flexural vibration of the bus-bar overhang, respectively. The free response of the SDOF model can be obtained as

\[ x(t) = X e^{-\zeta \omega_n t} \sin(\omega_d t + \phi) \]  

(A2)

where, \( \omega_n = \sqrt{k/m} \), \( \zeta = (c/(2\sqrt{km})) \) (to be estimated), and \( \omega_d = \omega_n \sqrt{1 - \zeta^2} \) have been defined in Section 2.1, and the amplitude \( X \) and phase \( \phi \) are obtained by imposing the initial conditions

\[ X = \sqrt{(x_0 + \omega_n \zeta x_0)^2 + (x_0 \omega_d)^2}, \quad \phi = \tan^{-1}\left( \frac{x_0 \omega_d}{x_0 + \omega_n \zeta x_0} \right) \]  

(A3)

Fig. A.2 shows a sample plot of the solution given by Eq. (A2). It is noted that the response of the system decays over time because of the presence of the viscous damping, and from Eq. (A2), is enveloped by the curve \( Xe^{-\zeta \omega_n t} \). Define the logarithm decrement as damping,

\[ \delta = \ln \left( \frac{x_1}{x_2} \right) \]  

(A4)

where \( x_1 = x(t) \) and \( x_2 = x(t + T_d) \), and \( T_d = \frac{2\pi}{\omega_n} \) is the period of oscillation of the damped system. In other words, \( \delta \) is the natural logarithm of the ratio of successive amplitudes of the response over one period. From Eq. (A2), the ratio of the amplitudes can be obtained as

\[ \delta = \frac{2\pi \zeta}{\sqrt{1 - \zeta^2}} \]  

(A5)

Fig. A.1. Single-degree-of-freedom vibration model with linear viscous damping.

Fig. A.2. Decay of the free response over time.
Inverting the above expression gives the damping ratio in terms of the measurable variable \( \delta \)

\[
\zeta = \frac{\delta}{\sqrt{4\pi^2 + \delta^2}}
\]  

Note that one can also obtain the following general relations for the logarithmic decrement

\[
\delta = \ln \left( \frac{x_k}{x_{k+1}} \right), \quad n\delta = \ln \left( \frac{x_k}{x_{k+n}} \right), \quad k = 1, 2, 3, \ldots
\]

The above relations allow a means of averaging the values of \( \delta \) from experimental data. In practice, one would estimate the free vibration amplitudes from the response curve and then average the calculated values of \( \delta \). It should be noted that the application of the logarithmic decrement formulas (A4) and (A7) is feasible only if the peaks in the response curve can be clearly and easily identified. The logarithmic decrement method can also be applied to the velocity data (laser vibrometer in the experiments collect velocity data). In other words, one can similarly define

\[
\delta = \ln \left( \frac{\nu_1}{\nu_2} \right)
\]  

and apply the same approach to determine the damping ratio as outlined above.

Experiments were set up to estimate the material damping characteristics of the bus-bar coupons. Response of the transverse vibration of the coupon was recorded near its bottom middle location by a laser vibrometer (laser head model: OFV-505). According to the logarithmic decrement method, the damping ratio can be estimated by examining the free response of the coupon. The free response refers to the vibration of the coupon immediately after the welding process is completed, during which residual welding energy in the coupon will be dissipated through material internal damping.

Fig. A.3(a) plots the velocity data of the flexural vibration of the bus-bar coupon obtained from one experiment. Fig. A.3(b) shows the free response of the coupon immediately after the welding process is completed. FFT of this decaying response shows that the fundamental period of oscillation is about 20 kHz with a super-harmonic at about 40 kHz, see Fig. A.4. The super-harmonic results in a modulated signal as seen in Fig. A.3(b), with the decaying response \( Xe^{-\zeta \omega t} \) governed by the lower dominant frequency (20 kHz). It is noted that Fig. A.4 is typical of a welding process, indicating that it is generally difficult to identify response peaks to effectively apply formulas (A4) or (A7) for accurate numerical estimations of the material damping ratio. In such cases, a computer code should be developed to extract the values of the peaks and then curve fit these experimental data to the exponential model \( x = a e^{bt} \) (the enveloped response is \( Xe^{-\zeta \omega t} \)) where the curve fitting parameter \( b = -\zeta \omega_2 \) with \( \omega_2 = 20 \text{ kHz} \) from which the damping ratio \( \zeta \) can be estimated.

In order to do the curve fitting, one must first determine the range of data to be considered and this should be examined for every set of data. The curve fitting procedure discussed above was applied to the data of Fig. A.4(b) data between \( t = 1.763 \) and 1.77. The curve
fitting result gives: $b = -3.869 \times 10^2$, with a goodness of fit $R$-squared value, $R^2 = 0.993$ ($R^2 = 1$ for perfect fit). The damping ratio is thus estimated from $-3.869 \times 10^2 = -\zeta \times (2\pi \times 20 \times 10^3)$ or $\zeta = 0.0031$. This is consistent with the published values for copper [25]. We assume that the modal damping ratios are equal in the numerical simulations.

References