CHAPTER 11

DYNAMICS AND VIBRATIONS OF BATTERY TABS UNDER ULTRASONIC WELDING

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Abstract

The effect of structural vibration of the battery tab on the required sonotrode force during ultrasonic welding is studied by applying a longitudinal vibration model for the battery tab. It is found that the sonotrode force is greatly influenced by the kinetic properties, quantified by the equivalent mass, equivalent stiffness, and equivalent viscous damping, of the battery tab and cell pouch interface. This study provides a fundamental understanding of battery tab dynamics during ultrasonic welding and its effect on weld quality, and thus provides a guideline for design and welding of battery tabs.

The effects of longitudinal and flexural vibrations of the battery tab during ultrasonic welding on the development of axial normal stresses that occasionally cause cracks near the weld area are studied by applying a continuous vibration model. Analysis results show that fracture could occur near the weld area, due to low cycle fatigue as a result of large dynamic stresses induced by resonant flexural vibration of the battery tab during welding. The axial normal stresses due to longitudinal waves traveling along the battery tab are shown to be insignificant compared to those due to flexural waves as the longitudinal wavelength at a typical ultrasonic welding frequency (e.g., 20 kHz) is much larger than the battery tab length while the flexural wavelength is much shorter.

It has been observed that sufficient energy is required to produce proper bonding of battery tabs, while excessive energy can cause quality issues such as weld fracture and perforation. Therefore, it is important to have a product/process design in ultrasonic welding to ensure efficient energy conversion from ultrasonics to welding energy, minimizing energy loss in the process. Vibrational energy loss due to material damping of the Cu coupon during ultrasonic welding is discussed, where the material damping is modeled as Kevin-Voigt damping and determined experimentally. It is shown that substantial energy loss can occur during welding due to the flexural vibration of the Cu coupon, especially when the overhang (the upper part of the Cu coupon extended from the anvil) of the Cu coupon resonates at or close to the welding frequency, degrading the weld quality of battery tabs.

Keywords: Ultrasonic welding, Battery tabs, Dynamics of battery tabs, Vibration of battery tabs, Vibrational energy loss

11.1 Introduction

Ultrasonic metal welding for battery tabs must be performed with 100% reliability in battery pack manufacturing as the failure of one weld essentially results in a battery that is inoperative or cannot deliver the required power due to the electrical short caused by the failed weld. Moreover, this stringent weld quality control is of great concern for battery pack manufacturers as automotive batteries are exposed to harsh driving environment such as vibration, severe temperature, and possibly crash, all of which can affect battery performance and safety. Therefore, one of the main issues arising in ultrasonic welding of battery tabs is to ensure consistent weld quality that meets design specifications such as electrical conductivity and shear strength of the weld. The quality of ultrasonic metal welds depend on a number of factors such as...
weld configuration, mechanical and metallurgical properties of weld parts, and weld process parameters – weld power, time, frequency, amplitude, clamping pressure, welding temperature, etc. [1].

Understanding that vibration is the driving mechanism and also, at the same time, the cause of weld problems in USMW, the main objective of the present chapter is to introduce and discuss the fundamental aspects of dynamics and vibrations involved in ultrasonic welding of battery tabs. The present work is motivated by numerous laboratory tests which shows a significant variation in weld quality of battery tabs caused by a slight change in geometry and structural properties of the weld part that alter the vibration characteristics of the system.

This chapter presents three main topics. In Section 11.2, the dynamic response of the battery tab during ultrasonic welding and its effect on the sonotrode force required for welding are presented. Section 11.3 discusses the effects of geometry, dimensions, and boundary conditions of the battery tab on the development of dynamic stresses induced by the longitudinal and flexural vibrations of the battery tab during ultrasonic welding. In Section 11.4, vibrational energy loss associated with the longitudinal and flexural vibrations of the copper coupon during ultrasonic welding is studied.

11.2 Dynamics of Battery Tabs Under Ultrasonic Welding

Combinations of these factors determine the sonotrode force which is required to cause the shearing motion at the weld interface for solid-state bonding. It should be noted that if the required sonotrode force for welding is larger than the gripping force of the sonotrode tip (welding tip), the sonotrode tip will slide against the weld part, resulting in extrusion or even no welding. Note that the gripping force of the sonotrode tip is traction at the interface between the sonotrode weld tip and weld part which solely depends on the size and knurl pattern of the sonotrode tip and the clamping pressure. Therefore it is a prerequisite for USMW that the required sonotrode force for welding should be as small as possible and must not exceed the gripping force of the sonotrode tip during the weld cycle [2].

The sonotrode force required for welding is a resultant force of the inertia force of the weld spot element (weld part pressed by the sonotrode tip) and the elastic/plastic friction force at the weld interface. The sonotrode force must be larger than this resultant force to induce a shearing motion at the weld interface for welding. For a weld part whose size is not significantly larger than the size of weld area, e.g., electrical contact pads or thin wires, the weld part may be considered as a rigid body since the entire weld part oscillates in phase with the sonotrode tip. However, when the dimensions of weld part is significantly larger than those of the weld area, e.g., spot welding of thin wall sections, the elastic vibrations of the weld part during welding should be taken into account for the determination of the upper limit of required sonotrode force. When the wavelength of ultrasonic excitation is comparable to the vibrational wavelengths of the weld part itself, the elastic vibrations of the weld part during welding may interact with the weld spot element causing the required sonotrode force to fluctuate beyond the maximum gripping force of the sonotrode tip. For the present ultrasonic welding of battery tabs, since the dimensions of the battery tab are much larger than the dimensions of the weld area, the structural vibrations of battery tabs are expected to play an important role in determining the weld quality by constantly changing the required sonotrode force during welding.

While a significant amount of research work on USMW and its applications has been made, most efforts have been focused on the aspects of weld metallurgy and weldability of different materials, however, there is only a limited amount of work to understand the overall dynamics of the ultrasonic welding system, particularly including the structural vibrations of weld parts and supporting structures (tools and fixtures). Jagota and Dawson [3] presented experimental and finite element analyses showing that the bonding strength of thin-walled thermoplastic parts by ultrasonic welding is strongly influenced by the lateral vibration of the weld parts. The impact of waveform designs, by controlling the wavelength of the ultrasonic input, on vibration response reduction of weld parts for the battery welding system is studied by Lee at al. [4].

1 The content presented in this section has previously appeared in reference [10].
The main objective of the present study is to examine the dynamic response of the battery tab during ultrasonic welding and assess its effect on the sonotrode force required for welding. This study is motivated by preliminary laboratory tests which show a significant variation in weld strength of battery tabs resulted from a slight alteration in structural properties of the weld part such as boundary conditions of the battery tab or anvil rigidity. A brief discussion on the free and forced longitudinal vibration of a thin bar is presented in Section 11.2.1 as the battery tab is modeled as a thin bar extended in the direction parallel to the excitation direction of the sonotrode. In Section 11.2.2, the tab-end force which is part of the required sonotrode force due to the elastic vibration of the battery tab is calculated for different end conditions of the battery tab. Experimental results on the kinetic properties of the tab-end are presented in Section 11.2.3. Summary and conclusions are presented in Section 11.2.4.

11.2.1 Theory and Modeling

In the present study, the battery tab is modeled as a thin bar extended parallel to the direction of ultrasonic excitation of the sonotrode, based on the fact that thickness of the battery tab is much smaller than other dimensions, particularly the longitudinal dimension, and on the assumption that the shear stresses developed in the weld spot element during welding result in a body force distributed over the weld spot. A brief introduction to the underlying theory applied to the longitudinal vibration analysis of the battery tab is presented in this section.

**Longitudinal Vibration of a Thin Bar**

Consider a thin, infinitely long, straight bar with a uniform cross-section subjected to an arbitrarily distributed axial body force \( p(x, t) \) (measured as a force per unit length) as shown in Figure 11.1. The equation governing the longitudinal vibration of the bar can be found as [5]:

\[
EA \frac{\partial^2 u}{\partial x^2} + p(x, t) = \rho A \frac{\partial^2 u}{\partial t^2}
\]  

(11.2.1)

where \( u = u(x, t) \) denotes the axial displacement of a cross-section, \( x \) the spatial coordinate, \( t \) the time, \( E \) the Young’s modulus, \( A \) the cross-sectional area, and \( \rho \) the mass density of the bar. In the absence of the body force, Eq. (11.2.1) reduces to the classical wave equation:

\[
\frac{\partial^2 u}{\partial x^2} = \frac{1}{c_0^2} \frac{\partial^2 u}{\partial t^2} \quad c_0 = \sqrt{\frac{E}{\rho}}
\]  

(11.2.2)

<table>
<thead>
<tr>
<th>Material</th>
<th>Mass density ( \rho ) [kg/m(^3)]</th>
<th>Young’s modulus ( E ) [GPa]</th>
<th>Phase velocity ( c_0 ) [m/s]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Aluminum</td>
<td>2,700</td>
<td>70</td>
<td>5,092</td>
</tr>
<tr>
<td>Copper</td>
<td>8,940</td>
<td>110</td>
<td>3,508</td>
</tr>
</tbody>
</table>

Figure 11.1 Slender bar with coordinate \( x \) and displacement \( u \).
where \( c_0 \) is the phase velocity (or \textit{bar velocity}) at which longitudinal waves propagate. Typical phase velocities in most metals are quite high compared to the velocity of sound in air of 340 m/s. Table 11.1 shows the phase velocities for the battery tab materials.

**Longitudinal Vibration of a Thin Bar with a Finite Length**

The solution of Eq. (11.2.1) can be found by assuming that

\[
u(x, t) = U(x)G(t) \tag{11.2.3}
\]

where

\[
U(x) = C_1 \cos \gamma x + C_2 \sin \gamma x \tag{11.2.4}
\]

\[
G(t) = D_1 \cos \omega t + D_2 \sin \omega t \tag{11.2.5}
\]

where the radial frequency \( \omega \), longitudinal wavenumber \( \gamma \), and longitudinal wavelength \( \lambda \) (the distance between two successive points of constant phase) are related by

\[
\omega = c_0 \frac{\gamma}{\lambda} \tag{11.2.6}
\]

The arbitrary constants in Eqs. (11.2.4) and (11.2.5) depend on the boundary and initial conditions. For example, consider a bar free at one end \((x = 0)\) and fixed at the other end \((x = L)\). The free boundary condition at \(x = 0\) implies that the stress at the bar end must be zero, therefore

\[
E \frac{\partial u(0, t)}{\partial x} = E \frac{dU(0)}{dx} G(t) = E \gamma C_2 G(t) = 0 \tag{11.2.7}
\]

Since \( G(t) \neq 0 \) and \( \lambda \neq 0 \), Eq. (11.2.7) dictates \( C_2 = 0 \). The fixed boundary at \(x = L\) requires that

\[
u(L, t) = U(L)G(t) = C_1 \cos \gamma L G(t) = 0 \tag{11.2.8}
\]

Since \( C_1 \neq 0 \), Eq. (11.2.8) yields

\[
\cos \gamma L = 0 \tag{11.2.9}
\]

which is the frequency equation for the free-fixed bar. Eq. (11.2.9) is satisfied only when

\[
\gamma_n = \frac{(2n - 1)\pi}{2L} \quad n = 1, 2, 3, \ldots \tag{11.2.10}
\]

Thus, the longitudinal natural frequencies of the bar system can be found from

\[
\omega_n = \frac{(2n - 1)\pi}{2L} c_0 \tag{11.2.11}
\]
where $\omega_n$ represents the discrete frequency at which the bar system undergoes harmonic motion. For a given value of $n$, the vibrational pattern (called the $n^{th}$ normal mode or modeshape function) of the bar is described by

$$U_n(x) = \cos \gamma_n x \quad (11.2.12)$$

Combining the time and spatial dependence for a given $n$, the assumed solution in Eq. (11.2.3) becomes

$$u_n(x,t) = (D_{1n} \cos \omega_n t + D_{2n} \sin \omega t) \cos \gamma_n x \quad (11.2.13)$$

The general solution to Eq. (11.2.2) is then obtained by superposing all normal mode solutions as

$$u(x,t) = \sum_{n=1}^{\infty} u_n(x,t) = \sum_{n=1}^{\infty} (D_{1n} \cos \omega_n t + D_{2n} \sin \omega t) \cos \gamma_n x \quad (11.2.14)$$

where the coefficients $D_{1n}$ and $D_{2n}$ are to be determined by applying the given initial conditions.

**Frequency Response Analysis**

As a simple example, consider the case of a bar, free at $x = 0$ and fixed at $x = L$, subjected to a harmonic end force $p(t) = p_0 \sin \Omega t$ at $x = 0$. Assuming the bar is initially at rest, the frequency response solution of the bar can be obtained by assuming a solution of the form

$$u(x,t) = U(x) \sin \Omega t \quad (11.2.15)$$

where $U(x)$ is given in Eq. (11.2.4). Applying the boundary conditions at $x = 0$ and $x = L$; i.e.,

$$EA \frac{dU(0)}{dx} = -p_0 \quad U(L) = 0 \quad (11.2.16)$$

the coefficient $C_1$ and $C_2$ in $U(x)$ can be found as

$$C_1 = \frac{p_0}{EA} \tan \gamma L \quad C_2 = -\frac{p_0}{EA} \quad (11.2.17)$$

The resulting forced longitudinal motion of the bar is

$$u(x,t) = \frac{p_0}{EA} (\tan \gamma L \cos \gamma x - \sin \gamma x) \sin \Omega t \quad (11.2.18)$$

It can be seen that the response becomes unbounded at the frequencies corresponding to $\cos \gamma L = 0$, or

$$\Omega = \frac{(2n - 1)\pi}{2L} c_0 \quad n = 1,2,3,\ldots \quad (11.2.19)$$

**11.2.2 Dynamics of the Battery Tabs**

Shown in Figure 11.2 is the cross-sectional view of a single battery cell assembly, where the battery tabs and bus-bar are clamped between the sonotrode tip and anvil. For the present study, noting that the thickness of the tab is much smaller than the other dimensions of the tab, the tab is modeled as a thin bar.
under longitudinal (x-direction) vibration subjected to boundary excitation due to the oscillatory motion of the weld spot element, based on the following observations and assumptions:

1) Only the top tab that is in contact with the sonotrode weld tip is considered in the present study. The coupling through friction and shearing motion at the weld interface between the top tab and the subsequent tab is represented by the interface force \( F_i \) (see Figure 11.2). Note that \( F_i \) is the resulting effect of the dynamic behavior of other weld parts below the top tab during welding.

2) No (or negligible) elastic wave motion within the weld spot element beneath the sonotrode tip is possible since the weld spot element is pressed and constrained by the knurl-patterned sonotrode tip. This implies that the weld spot element can be considered as a rigid body oscillating in phase with the sonotrode tip, which is the source of longitudinal excitation to the extended part of the tab.

3) The sonotrode force, the driving force acts upon the top tab (\( F_s = a \sin \omega t \) in Figure 11.2), is sufficiently larger than the interface force \( F_i \), otherwise extrusion or no welding occurs.

4) The sonotrode oscillates in the x-direction only, and its amplitude remains constant during welding; transverse (z-direction) vibration does not exist.

5) For the modeling purpose, a term tab-end is defined. As indicated in Figure 11.2, the tab-end includes part of the tab extended from the bend line and some part of the battery cell pouch that surrounds the inserted tab.

![Figure 11.2 Schematic of the battery cell assembly (with the cell pouch partially shown).](image1)

![Figure 11.3 Free body diagram for the weld spot element and coordinate system.](image2)
Note that the shearing motion of the weld spot element in the tab during welding depends on not only the sonotrode force and friction at the weld interface but also the elastic vibration of the tab. The vibration characteristics of the tab is governed by the boundary conditions of the tab as discussed in Section 11.2.1, then it can be seen from Figure 11.2 that the tab-end constitutes a natural (kinetic) boundary condition for the tab. During ultrasonic welding, part of the vibration energy injected by the oscillating sonotrode tip travels along the tab, through the tab-end, and then eventually dissipates in the battery cell pouch which contains viscoelastic materials. Hence, the kinetic properties of the tab-end become an important factor determining the longitudinal vibration characteristics of the tab during welding. The kinetic properties of the tab-end are represented by the equivalent mass ($m_{eq}$), equivalent stiffness ($k_{eq}$), and equivalent viscous damping ($k_{eq}$) as shown in Figure 11.3. Due to complex geometry and material properties of the tab-end which consists of both parts of the battery tab and battery cell pouch, the determination of the equivalent mass, stiffness, and damping of the tab-end by analytical or numerical methods seems limited. An experimental dynamic test to measure the equivalent mass is outlined in Section 11.2.3. Note that the equivalent stiffness of the tab-end can be readily measured through a simple tensile test of the battery tab and cell pouch assembly.

Shown in Figure 11.3 is the free body diagram for the weld spot element in a battery cell sketched in Figure 11.2, subjected to three forces: sonotrode force $F_s$, interface force $F_i$ from the neighboring tab, and tab-end force $F_e = F_{e1} + F_{e2}$ which is due to the elastic vibration of the extended part of the tab during welding. From the free body diagram for the weld spot element in Figure 11.3, one can find that the minimum sonotrode force $F_s$ required for welding, i.e.,

$$F_s = m\ddot{x} + F_i + F_e$$  \hspace{1cm} (11.2.20)

The first term on the right side of Eq. (11.2.20) is the inertia force of the weld spot element due to the vibration of the sonotrode. Assuming that the sonotrode maintains its grip against the weld spot element during welding and that the sonotrode oscillates at the frequency $f$ [Hz] with the amplitude of $a$, i.e., $\alpha \sin \Omega t$, it can be found that

$$m\ddot{x} = -a\Omega^2 \sin \Omega t \quad \Omega = 2\pi f$$  \hspace{1cm} (11.2.21)

It is not an easy task to quantify the interface force $F_i$. This force is expected to be significantly larger than the other forces in Eq. (11.2.20). Note that due to the transitional behavior of friction migrating from dry to viscous friction as welding progresses, $F_i$ is not constant. Quantification of $F_i$ is not a trivial task and may require rigorous theoretical, numerical, and experimental analyses, and thus it is beyond the scope of the present study and left as future work. However, assuming that the entire weld interface is plastically yielded (i.e., ideal full metal-to-metal contact), one may theoretically approximate the maximum value of $F_i$ as a force that shears the weld. By applying the Tresca maximum-shear yield criterion for the two dimensional stress state [2] and noting that $\sigma_Y \gg p$, the theoretical maximum of $F_i$ can be found to be

$$\max F_i = \frac{A_p}{2} \sqrt{\sigma_Y^2 - p^2} \cong 0.5\sigma_Y A_p$$  \hspace{1cm} (11.2.22)

where $A_p$ is the area of plastic deformation zone (weld area) at the weld spot, $p$ the clamping pressure, and $\sigma_Y$ the yield strength of the tab material. More comprehensive discussion regarding the transitional behavior of the friction coefficient in USMW can be found in the study by Gao and Doumanidis [6].

The tab-end force $F_e$ acting on the weld spot element during welding can be determined by the boundary value analysis of the tab under longitudinal vibration. It is shown in the present study that the tab-end force $F_e$ can be significantly large and very sensitive to the effective mass $m_{eq}$ of the tab-end due to high
acceleration (over 16,000G at 20 kHz with sonotrode amplitude of 10 µm) during welding. A detailed analysis of the tab-end force is as follows.

**Natural Frequency Analysis of the Battery Tab**

When the wavelength of ultrasonic excitation is comparable to the vibrational wavelengths of the weld part itself, the weld part may be induced to vibrate by the ultrasonic welding system; i.e., resonance can occur. This resonance could cause inconsistent weld quality or a structural failure of the weld part. In order to examine possible resonance of the tab during welding, the natural frequencies of the tab are determined and compared with the ultrasonic welding frequency. With reference to Figure 11.3, the boundary conditions for the tab are

\[
U'(0) = 0 \quad U'(L) = \frac{m_{eq} \omega^2 - k_{eq} U(L)}{EA} 
\]

where \( L \) is the tab length, i.e., \( L = L_1 + L_2 + b \). Applying the above boundary conditions to Eq. (11.2.4), it can be found that the natural frequencies of the tab must satisfy the following frequency equation

\[
m_{eq} \beta_n^2 + \beta_n \tan \beta_n - \hat{k}_{eq} = 0 \quad \beta_n = \frac{L}{c_0 \omega_n} 
\]

where \( \beta_n \) denotes the wavenumber (number of repeating waves) for the \( n \)th vibration mode and \( \hat{m}_{eq} \) and \( \hat{k}_{eq} \) are the nondimensional equivalent mass and stiffness of the tab-end, respectively, normalized by

\[
\hat{m}_{eq} = \frac{m_{eq}}{\rho AL} \quad \hat{k}_{eq} = \frac{k_{eq} L}{EA} 
\]

![Figure 11.4 Wavenumber loci of the first 6 longitudinal vibration modes. \( \hat{k}_{eq} = 5.23 \times 10^{-3} \).](image)

The frequency equation in Eq. (11.2.24) needs to be solved numerically, hence \( k_{eq} = 150 \) kN/m is assumed for both aluminum and copper tabs, which gives \( \hat{k}_{eq} = 5.23 \times 10^{-3} \) for the aluminum tab and \( \hat{k}_{eq} = 3.33 \times 10^{-3} \) for the copper tab. Assuming \( L = 20 \) mm, \( 0 \leq \hat{m}_{eq} \leq 2 \) is considered for numerical simulations, which corresponds to \( 0 \leq m_{eq} \leq 0.89 \) gram for the aluminum tab and \( 0 \leq m_{eq} < 2.93 \) gram.
for the copper tab. Shown in Figure 11.4 are the wavenumber loci as a function of $\hat{m}_{eq}$ for the first six longitudinal vibration modes of the tab. Notable findings are as follows.

- The wavenumber of the fundamental longitudinal vibration mode of the tab is very small. For example, $\beta_1 = 0.07$ for $\hat{m}_{eq} = 0$ and $\beta_1 = 0.04$ for $\hat{m}_{eq} = 2$, each corresponding to the wavelength of 1,795 mm and 3,142 mm. This suggests that the fundamental longitudinal vibration mode of the tab behaves almost like a rigid body mode.
- The effect of increasing $\hat{m}_{eq}$ on the longitudinal wavenumbers of the tab becomes quickly saturated for all vibration modes.
- Although not presented, under the presence of $\hat{m}_{eq}$, $\hat{k}_{eq}$ has an insignificant effect on altering the wavenumbers of the tab for all vibration modes unless it is very large. Note that the fundamental wavenumber is $\pi/2$ when $\hat{k}_{eq} = \infty$.

Shown in Figure 11.5 are the natural frequency loci of the longitudinal vibration for each tab against the equivalent mass of the tab-end when $k_{eq} = 150$ kN/m. It can be noticed that the current ultrasonic welding frequency (20 kHz) is not close to any of the natural frequencies for both aluminum and copper tabs, regardless of $m_{eq}$, indicating little possibility for resonance of the tab during welding.

![Figure 11.5](image-url)

**Figure 11.5** Longitudinal natural frequency loci up to 400 kHz against $m_{eq}$ for $L = 20$ mm and $k_{eq} = 150$ kN/m. The dashed line represents the ultrasonic welding frequency $\Omega$. 
Dynamic Effects of the Tab-End

Ultrasonic weld quality of battery tabs depends on the mechanical properties of the battery tab itself including its geometry/dimensions in addition to weld parameters such as weld frequency, clamping pressure, pre-weld surface condition, and weld time. While the thickness of the individual battery tabs is an important factor that determines the weldability of multiple battery tabs and bus-bar, the weld quality (weld strength in general) depends on the size of the tab relative to the weld spot area, especially the dimension of the tab in the direction of excitation. If this dimension is comparable with the ultrasonic wavelength, the elastic vibration of the tab in the direction of welding will significantly increase the sonotrode force required for welding, and could be detrimental to welding when resonance occurs. The elastic vibration of the battery tab and its resonant behavior are characterized by the length \((L_1\) in Figure 11.3) between the weld spot and tab-end as well as the kinetic properties of that tab-end. Under the current welding practice of battery tabs, \(L_1\) varies depending on the location of weld spot.

A typical ultrasonic metal weld cycle is less than 1 second. It should be noted that ultrasonic weld process is transient by its nature as the friction/shearing force at the weld interface is not steady, but usually increases as bonding areas at the weld interface increase. Nevertheless, the concept of steady state can be applied to the present tab-end force analysis for the following two reasons; (1) the sonotrode of ultrasonic welder is under feedback control to maintain its vibration amplitude constant during welding and (2) the sonotrode force (that grabs and excites the weld part) is larger than the sum of other resistant forces (inertia force of weld element, interface force, and tab-end force), otherwise the sonotrode tip loses its grip against the weld part and extrusion or no welding occurs. Therefore, the weld spot element which is pressed and constrained by the sonotrode tip can be modeled as a rigid body oscillating in phase with the sonotrode tip. This rigid body motion of the weld spot element acts as sinusoidal boundary excitation to the extended part of the battery tab. Various experiments (not shown in the present study) show that the sonotrode weld tip reaches its steady state at the very beginning of the weld cycle with the weld frequency of 20 kHz.

In order to determine the tab-end forces \(F_{e1}\) and \(F_{e2}\) acting on the weld spot element, the tab is divided into two segments with respect to the weld spot element, i.e., \(S_1\) segment \((0 \leq \xi_1 \leq L_1)\) which is on the right side of the weld spot element and \(S_2\) segment \((0 \leq \xi_2 \leq L_2)\) on the left side of the weld spot element as shown in Figure 11.3. To determine \(F_{e1}\), consider \(S_1\) segment of the tab. Since the weld spot element is rigid and oscillates with the sonotrode in the same phase, the velocity at \(\xi_1 = 0\) of \(S_1\) segment must be the same as the sonotrode tip velocity \(a\Omega\). Moreover, at the other end \((\xi_1 = L_1)\), \(S_1\) segment interacts with the tab-end. Therefore, the boundary conditions for \(S_1\) segment of the tab are:

\[
U(0) = a \left( \frac{dU(L_1)}{d\xi_1} \right) = \frac{k_{eq}L - ic_{eq}\Omega - k_{eq}}{EA} U(L_1) \quad (11.2.26)
\]

Applying the above boundary conditions to Eq. (11.2.4), the steady-state longitudinal displacement of \(S_1\) segment of the tab can be found as

\[
u(\xi_1, t) = a|H(\xi_1, i\Omega)| \sin(\Omega t - \phi) \quad 0 \leq \xi_1 \leq L_1 \quad (11.2.27)
\]

where \(H(\xi_1, i\Omega)\) is the complex frequency response function, defined by

\[
H(\xi_1, i\Omega) = \cos \left( \frac{\Omega}{c_0} \xi_1 \right) + \alpha(i\Omega) \sin \left( \frac{\Omega}{c_0} \xi_1 \right) \quad (11.2.28)
\]

\[
\alpha(i\Omega) = \frac{(m_{eq}\Omega^2 - ic_{eq}\Omega - k_{eq})}{EA} \cos \left( \frac{\Omega}{c_0} L_1 \right) + \frac{\Omega}{c_0} \sin \left( \frac{\Omega}{c_0} L_1 \right) \quad (11.2.29)
\]
and \( \phi \) the phase angle, defined by \( \phi = \angle H(\xi_1, i\Omega) \). If \( c_{eq} = 0 \), \( \alpha(i\Omega) = \tan((\Omega/c_0)L_1) \) when either \( \Omega = \sqrt{k_{eq}/m_{eq}} \) or \( m_{eq} = k_{eq} = 0 \) and that \( H(\xi_1, i\Omega) \) reduces to
\[
H(\xi_1, i\Omega) = \cos \left( \frac{\Omega}{c_0} \xi_1 \right) + \tan \left( \frac{\Omega}{c_0} L_1 \right) \sin \left( \frac{\Omega}{c_0} \xi_1 \right) \quad (11.2.30)
\]
This implies, when \( c_{eq} = 0 \) and \( \Omega = \sqrt{k_{eq}/m_{eq}} \), the dynamic effect of the tab-end is identical to the one with the free boundary condition. For \( S_2 \) segment of the tab, since \( m_{eq} = k_{eq} = c_{eq} = 0 \), it can be readily found that
\[
u(\xi_1, t) = a \left( \cos \left( \frac{\Omega}{c_0} \xi_2 \right) + \tan \left( \frac{\Omega}{c_0} L_2 \right) \sin \left( \frac{\Omega}{c_0} \xi_2 \right) \right) \sin \Omega t \quad 0 \leq \xi_2 \leq L_2 \quad (11.2.31)
\]

### Axial Stress Distribution in the Tab

Since \( \sigma_x = E \frac{\partial u}{\partial x} \), the axial stress distribution in each segment of the tab can be found from Eqs. (11.2.27) and (11.2.31). For \( S_1 \) segment,
\[
\sigma_x(\xi_1, t) = Ea \left| \frac{\partial H(\xi_1, i\Omega)}{\partial \xi_1} \right| \sin(\Omega t - \phi) \quad 0 \leq \xi_1 \leq L_1 \quad (11.2.32)
\]
and for \( S_2 \) segment
\[
\sigma_x(\xi_2, t) = Ea \frac{\Omega}{c_0} \left( \tan \left( \frac{\Omega}{c_0} L_2 \right) \cos \left( \frac{\Omega}{c_0} \xi_2 \right) - \sin \left( \frac{\Omega}{c_0} \xi_2 \right) \right) \sin \Omega t \quad 0 \leq \xi_2 \leq L_2 \quad (11.2.33)
\]

Figure 11.6 shows the axial stress distributions in \( S_1 \) segment of the tab for different values of \( m_{eq} \), where \( L_1 = 20 \) mm and \( k_{eq} = 150 \) kN/m and \( c_{eq} = 0 \). The cases for free \( (m_{eq} = k_{eq} = c_{eq} = 0) \) and fixed \( (k_{eq} = \infty \) and \( m_{eq} = c_{eq} = 0) \) boundary conditions are also shown as the limiting cases. Notable behavior is summarized as follows:

- Stress distributions in the tab are monotonic with a gradual decrease in slopes toward the tab-end, indicating that the stress wavelength is much larger than the tab length.
- Depending on the equivalent mass of the tab-end, the entire or part of the tab can be subjected to dynamic stresses exceeding the material’s yield strength \( (\sigma_Y = 55 \) MPa for aluminum and \( \sigma_Y = 172 \) MPa for copper at 25°C). These high stresses could plasticize the tab material and make the tab prone to buckling during welding under certain conditions, for example the transverse vibration of the tab or material irregularity.
- Large stresses in the tab during welding may be indicative of the loss of welding energy. In other words, part of the welding energy gives rise to increase in the overall strain energy of the tab. It is necessary to employ a design to minimize the equivalent mass (or its effect) of the tab-end.
- As previously mentioned, when \( \Omega = \sqrt{k_{eq}/m_{eq}} \) and \( c_{eq} = 0 \), the tab-end behaves as if it is free of constraints. This fact could be utilized for the design of tab-pouch interface to lower the stresses in the tab during welding.
- The effect of the equivalent stiffness of the tab-end is not as drastic as the equivalent mass. This can be inferred by comparing the stress distribution curves between the two extreme cases, free and fixed boundary conditions. It can be seen that the difference in stresses is relatively small, even between these two extreme cases, indicating that the dynamics of the tab during welding is more affected by the equivalent mass rather than the equivalent stiffness.
Figure 11.6 Axial stress distribution in the tab due to longitudinal vibration of the tab with $k_{eq} = 150$ kN/m, where $L_1 = 20$ mm and the numbers indicate $m_{eq}$ in grams.

**Effect of Weld Spot Location on the Tab-End Force**

The weld spot location ($L_2$ in Figure 11.3) is an important design parameter for battery tab welding as it has been observed in various experiments that a slight change (less than 1 mm) of the weld spot location causes excessive vibration of the tab during welding, resulting in unacceptable weld strength or extrusion due to increase of sonotrode force required for welding. From Eq. (11.2.32), the tab-end force $F_{e1} = A\sigma_x(0,t)$ exerting on the weld spot element due to the elastic vibration of $S_1$ segment of the tab can be found as

$$F_{e1} = EAa \left| \frac{\partial H(0,i\Omega)}{\partial \xi_1} \right| \sin(\Omega t - \phi)$$  \hspace{1cm} (11.2.34)

In a similar manner, from Eq. (11.2.33), the tab-end force exerted by $S_2$ segment of the tab is

$$F_{e2} = EAa \frac{\Omega}{c_0} \tan \left( \frac{\Omega L_2}{c_0} \right) \sin \Omega t$$  \hspace{1cm} (11.2.35)
Figure 11.7 Tab-end force as a function of the weld spot location with \( k_{eq} = 150 \text{ kN/m} \), where the total length of the tab is \( L = 20 \text{ mm} \) and the numbers indicate \( m_{eq} \) in grams.

The total amplitude of the tab-end force acting on the weld spot element becomes \( F_e = F_{e1} + F_{e2} \). It can be seen from Eqs. (11.2.34) and (11.2.35) that the tab-end force acting on the weld spot element depends on the span length of each segment as well as the equivalent mass, stiffness, and damping of the tab-end. In other words, the location of the weld spot relative to the entire tab length also affects the sonotrode force required for welding. Figure 11.7 shows the total tab-end force \( F_e \) acting on the weld spot element for each of the aluminum and copper tabs as a function of the weld spot location measured from the free end (i.e., \( x = 0 \) in Figure 11.3) of the tab, for slightly different values of the equivalent mass of the tab-end, demonstrating the effect of tab-end dynamics. Some notable behavior is summarized as follows:

- The weld spot location plays an important role in determining the tab-end force, and thus the sonotrode force required for welding. A slight change in the equivalent mass of the tab-end significantly changes the tab-end force.
- For the aluminum tab, the tab-end force is not a simple linear function of the equivalent mass. For example, when \( L_2 = 0 \), the smallest tab-end force is when \( m_{eq} = 0.5 \text{ gram} \). A similar behavior can be found for the copper tab, however in this case \( m_{eq} \) for the smallest tab-end force is much larger than the one for the aluminum tab.
- The tab-end force strongly depends on the elastic vibration of the tab. Since the elastic vibration of the tab depends on the mechanical properties (i.e., \( \rho \) and \( E \)), tab length, and boundary conditions
at the tab-end, depending on the combinations these parameters, the tab-end force may increase or
decrease as the weld spot location changes. In comparison of Figure 11.7(a) and Figure 11.7 (b),
if one shifts the curves in Figure 11.7 (b) toward the origin, it can be seen that the overall behavior
of the tab-end force for both aluminum and copper tabs is similar. In other words, the inflection
points of the individual curves for the copper tab are located at larger values of $L_2$. This is due to
the natural frequencies of the copper tab are lower than the ones of the aluminum tab. Note also
that as $L_2$ increases, i.e., as the weld spot location moves closer to the tab-end, the elastic vibration
of Segment 2 of the tab (see Figure 11.3) results in increasing the tab-end force.

- Although not shown in the plots, it is found that the effect of the equivalent stiffness of the tab-end
  on the tab-end force is not as drastic as the equivalent mass.
- The relation between the weld spot location and tab-end force may serve as a guideline for battery
  tab design and welding. For example, when $m_{eq} = 0.5$ gram for the aluminum tab, $L_2 = 1.1$ mm is
  the optimal Z-height for the minimum tab-end force.

![Figure 11.8 Tab-end force as a function of the weld spot location, where the total length of the tab is $L = 20$ mm. The numbers indicate $c_{eq}$ in [Ns/m]. (a) Aluminum tab, C-bend, $k_{eq} = 105$ kN/m, and $m_{eq} = 0.0063$ g and (b) copper tab, C-bend, $k_{eq} = 147$ kN/m, and $m_{eq} = 0.0102$ g.](image-url)
Figure 11.9  Energy dissipation per cycle by the equivalent viscous damper at the tab-end, as a function of tab length $L_1$. The numbers indicate $c_{eq}$ in [Ns/m]. (a) Aluminum tab, C-bend, $k_{eq} = 105$ kN/m, and $m_{eq} = 0.0063$ g and (b) copper tab, C-bend, $k_{eq} = 147$ kN/m, and $m_{eq} = 0.012$ g.

**Effect of Damping on the Tab-End Force and Energy Dissipation**

Shown in Figure 11.8 is the total tab-end force as a function of the weld spot location for several different values of the equivalent viscous damping coefficient, $c_{eq}$, at the tab-end. Note that the welding (excitation) frequency, $\Omega = 20$ kHz, is between the first and second natural frequencies, being closer to the first one, of both aluminum and copper tabs when $L_1 = 20$ mm. Note also that the critical modal damping coefficient for the first vibration mode is about 18 Ns/m for the aluminum tab with $k_{eq} = 105$ kN/m and $m_{eq} = 0.0063$ g, and 32 Ns/m for the copper tab with $k_{eq} = 147$ kN/m and $m_{eq} = 0.012$ g. It can be seen that the tab-end force increases with increasing $c_{eq}$. When damping is present, it is of more interest
to know the amount of vibration energy dissipated by the damping during welding. Denoting $\Delta E_d$ as the energy dissipation per cycle by the equivalent viscous damper at the tab-end (i.e., $\xi_1 = L_1$), it can be found that

$$\Delta E_d = \int_{-\phi/\pi}^{2\pi - \phi/\pi} c_{eq} \ddot{u}^2(L_1, t) dt = \pi c_{eq} \Omega^2 |H(L_1, i\Omega)|^2$$  \hspace{1cm} (11.2.36)

Figure 11.9 shows the plots of $\Delta E_d$ as a function of the tab length $L_1$ for different values of $c_{eq}$. Note that the steady state response amplitude of aluminum tab with $L_1 = 44$ mm, $k_{eq} = 105$ kN/m, and $m_{eq} = 0.0063$ g is largest as the excitation frequency $\Omega$ is close to the second damped natural frequency of the tab when $c_{eq} = 10$ Ns/m, and that the steady state response amplitude of copper tab with $L_1 = 44$ mm, $k_{eq} = 147$ kN/m, and $m_{eq} = 0.012$ gram is largest for the values of $c_{eq}$ considered in this numerical example. As expected, it can be seen that the energy dissipation by damping peaks at resonance. For example, when $c_{eq} = 10$ Ns/m, about 0.3 joule of energy is dissipated per each excitation cycle at resonance, which amounts to 3,000 joule of energy dissipation for 0.5 seconds steady-state welding time at $\Omega = 20$ kHz. Note that the power of the ultrasonic welder used for the current welding of battery tabs is 3,000 watts. However, a typical tab length ($L_1$) in the current practice of battery tabs welding is within 11 and 17 mm as indicated in the figure. In this range, the energy dissipated by the damping at the tab-end is of the order of $10^{-3}$ joule per cycle for the values of $c_{eq}$ considered in this example.

Sonotrode Force Required for Welding

Recalling Eq. (11.2.20), the required sonotrode force $F_s$ for welding is the sum of the three non-constant forces; inertia force $ma\Omega^2$ of the weld spot element, elastic/plastic friction force $F_i$ at the weld interface, and tab-end force $F_e$ due to the longitudinal vibration of the tab. It has been suggested by the present analysis that $F_i > F_e > ma\Omega^2$ in general. The interface force $F_i$ rapidly increases as welding progresses to its maximum value, inducing plastic deformation at the weld interface [6]. While $F_i$ is at its maximum, it is possible that the sum of the other two forces ($ma\Omega^2 + F_e$) causes the required sonotrode force to exceed its upper limit which is the gripping force ($F_g$) at the sonotrode-tab interface. Noted that $F_g$ is a constant force which depends solely on the clamping pressure and knurl pattern of the sonotrode tip. When $F_g > F_e$, the sonotrode tip loses its grip on the tab, which would result in extrusion or unacceptable welding. For welding to occur, the peak value of the required sonotrode force must not exceed the gripping force during the weld cycle. As demonstrated in the present analysis results, the tab-end force is significantly influenced by the longitudinal vibration of the tab itself which in turn depends on the kinetic properties of the tab-end, i.e., equivalent mass and stiffness. Therefore a proper design of the battery tab and cell pouch interface can minimize the tab-end force, thus lowering the required sonotrode force during welding.

11.2.3 Experimental Results and Discussion

Experimental Measurement of the Equivalent Mass and Stiffness of Tab-End

The equivalent stiffness of the tab-end can be readily measured through a typical tensile test. While the battery cell pouch and tab assembly is secured (by using a fixture) in the same manner as it is constrained in the battery module during welding, the tab is quasi-statically pulled by a tensile testing machine to generate a force-displacement curve, where note that grip on the tab must be right above the bend line. The maximum slope of the force-displacement curve is the measure of the equivalent stiffness of the tab-end. Figure 11.10(a) shows schematically the technique to measure the equivalent mass using an ultrasonic welder, a laser vibrometer with DAQ, and a dummy mass securely affixed to the battery tab. In addition, a fixture is required to clamp the battery cell pouch in the same manner as in actual welding. During welding, the dummy mass vibrates in response to the sonotrode excitation through the longitudinal motion of the tab.
Once the response amplitude of the dummy mass is measured with the laser vibrometer, the equivalent mass of the tab-end can be calculated from the sinusoidal transfer function of the equivalent 2-DOF mass-spring system shown in Figure 11.10(b). The equations of motion of the equivalent system are

\[ m\ddot{x} + (k + k_{eq})x = ku + k_{eq}y \]  \hspace{1cm} (11.2.37)

\[ m\ddot{y} + k_{eq}y = k_{eq}x \]  \hspace{1cm} (11.2.38)

Figure 11.10 (a) Experimental setup for measurement of the equivalent mass of the tab-end and (b) equivalent 2-DOF system.

Table 11.2 Equivalent stiffness \( (k_{eq}) \) of the tab-end

<table>
<thead>
<tr>
<th>Bend shape</th>
<th>Al-tab [kN/m]</th>
<th>Cu-tab [kN/m]</th>
</tr>
</thead>
<tbody>
<tr>
<td>C-bend ( )</td>
<td>94 – 115</td>
<td>114 – 180</td>
</tr>
<tr>
<td>S-bend ( )</td>
<td>21 – 42</td>
<td>33 – 53</td>
</tr>
</tbody>
</table>

Table 11.3 Equivalent mass \( (m_{eq}) \) of the tab-end.

<table>
<thead>
<tr>
<th>Bend shape</th>
<th>Al-tab [grams]</th>
<th>Cu-tab [grams]</th>
</tr>
</thead>
<tbody>
<tr>
<td>C-bend ( )</td>
<td>0.006 – 0.0066</td>
<td>0.0091 – 0.0114</td>
</tr>
<tr>
<td>S-bend ( )</td>
<td>0.0013 – 0.0027</td>
<td>0.0021 – 0.0034</td>
</tr>
</tbody>
</table>

where \( m \) is the mass of the dummy mass and \( k \) is the longitudinal stiffness of the tab between the weld spot and dummy mass as shown in Figure 11.10(a). From the above equations, the sinusoidal transfer function for the dummy mass can be found as

\[ G(i\omega) = \frac{ka}{(k + k_{eq}) - m\Omega^2 - \frac{k_{eq}^2}{k_{eq}^2 - m_{eq}^2\Omega^2}} \]  \hspace{1cm} (11.2.39)

Hence, the steady-state response amplitude of the dummy mass is

\[ X_{ss} = aG(i\omega) \]  \hspace{1cm} (11.2.40)
where $a$ is the sonotrode amplitude. Since $k_{eq}$ and $X_{eq}$ are known from the measurement, $m_{eq}$ can be found by solving Eq. (11.2.40) for $m_{eq}$. According to the methodologies described above, the equivalent mass ($m_{eq}$) and stiffness ($k_{eq}$) of the tab-end have been experimentally determined. For the measurement of $k_{eq}$, a single battery cell-pouch, insulation form, and cooling plates are placed between two nylon frames clamped by a specially built fixture in order to replicate the same boundary conditions for the battery cell-pouch as it is secured in the battery module during welding. Instron tensile testing machine with DAQ is used to obtain the $p-\delta$ curve for each of the $C$-bend and $S$-bend tabs, from which $k_{eq}$ of the tab-end is obtained and summarized in Table 11.2.

![Figure 11.11 Velocity ($\dot{X}_{45^\circ}$) of the dummy mass for aluminum tab.](image-url)
In order to determine \( m_{eq} \) of the tab-end, the velocity \( (\dot{X}_{45\degree}) \) of the dummy mass is measured at 45\(^\circ\) (due to interference with the fixture and welder) by using the Polytec laser vibrometer, and from which the velocity \( (\dot{X}) \) in the weld direction can be found by \( \dot{X} = \sqrt{2}\dot{X}_{45\degree} \). Figures 11.11 and 11.12 show the measured velocity \( (\dot{X}_{45\degree}) \) of the dummy mass. Applying \( m = 4.7 \) grams, and \( k_{eq} \), and the steady-state velocity amplitude for each tab to Eq. (11.2.40), \( m_{eq} \) of the tab-end is determined as summarized in Table 11.3. It can be seen that \( m_{eq} \) of the tab-end is found to be insignificantly small. It is believed that the tab-bend effectively weakens the dynamic coupling between the battery tab and cell-pouch.

**Dynamic Effects of the Tab-End**

By applying the measured values of the effective mass and stiffness of the tab-end, the axial stress
distribution in the tab during welding and the resulting tab-end force are computed. Shown in Figure 11.13 is the axial stress distribution of the tab (Segment 1). It can be seen that axial stresses are well below the yield strength (55 MPa for Al-tab and 172 MPa for Cu tab at 25°C) and very little differences in stresses between C-bend and S-bend. Figure 11.14 shows the tab-end force as a function of the weld spot location. It can be seen that 1 mm change in the Z-height toward the battery cell-pouch lowers the tab-end force by about 1 N for Al-tab and 5 N for Cu-tab within a practical range of weld spot location. Note that the range of the weld spot location in the current practice is between 0 and 2 mm.

11.2.4 Summary and Conclusions

The effect of dynamic response of a single battery tab on the sonotrode force required for welding is studied by applying a one-dimensional continuous vibration model for the battery tab. The battery tab is modeled as a thin bar vibrating longitudinally under ultrasonic excitation from the sonotrode. This study serves as the foundation for a scientific understanding of battery tab dynamics during ultrasonic welding and its effect on weld quality, and thus provides a guideline for design and welding of battery tabs. Notable findings are summarized as follows:

1) A slight change in the kinetic properties of the battery tab-end (interface between the tab and battery cell pouch), being amplified by the longitudinal vibration of the battery tab at high acceleration during ultrasonic welding, causes a significant change in the sonotrode force required for welding. Experimental quantification of the kinetic properties of the tab-end in terms of the equivalent mass, stiffness, and viscous damper as key design parameters is needed for the design of battery tabs to improve the weld quality.

2) Axial stresses of the battery tab during ultrasonic welding could exceed the material’s yield strength when the equivalent mass of the tab-end is large, suggesting that the battery tab is prone to plastic deformation and buckling due to dynamic instability triggered by subtle transverse motions such as anvil or bus-bar vibrations. Reduction of the equivalent mass of the battery tab-end can lower the required sonotrode force for welding.

3) The difference in sonotrode forces required for welding between the aluminum and copper tab is significantly large. That is, the sonotrode force required for welding of the aluminum tab is significantly lower than welding the copper tab. Studies on the effect of an excessive sonotrode force on weld quality is warranted.

4) The sonotrode force required for welding is substantially affected by the weld spot location. The optimal location of weld spot for the minimum sonotrode force also depends on the kinetic properties of the battery tab end.
Figure 11.13 Axial stress distribution due to longitudinal vibration of the tab, where \( L_1 = 20 \) mm.
Figure 11.14 Tab-end force against weld spot location, where the total length of the tab is $L = 23$ mm.
Nomenclature

\( A \)  Cross-sectional area of battery tab (45mm × 0.2mm)
\( a \)  Sonotrode amplitude (20 \( \mu \)m peak-to-peak)
\( A_p \)  Weld area (= sonotrode tip areas, 3×(3 mm×5 mm) = 45 mm²)
\( b \)  Longitudinal dimension of weld spot (3 mm is used for numerical examples)
\( c_0 \)  Phase velocity \( (= \sqrt{E/\rho}) \)
\( c_{eq} \)  Equivalent viscous damping coefficient (see Figure 11.3)
\( E \)  Young’s modulus (70 GPa for aluminum and 110 GPa for copper)
\( F_s \)  Sonotrode force [N]
\( F_e \)  Tab-end force [N] \( (F_e = F_{e1} + F_{e2}, \text{see Figure 11.3}) \)
\( F_i \)  Interface force at weld spot [N]
\( k_{eq} \)  Equivalent stiffness of the tab-end (see Figure 11.3)
\( L \)  Tab length (see Figure 11.3)
\( \ell \)  Distance between the top of dummy mass and the bottom of sonotrode tip (see Figure 11.10)
\( L_1 \)  Length of the tab between the weld spot and tab-end (see Figure 11.3)
\( L_2 \)  Length of the tab between the weld spot and free end (see Figure 11.3)
\( m \)  Mass of the weld spot element [kg]
\( m_{eq} \)  Equivalent mass of the tab-end (see Figure 11.3)
\( u \)  Longitudinal displacement of the tab [mm]
\( U_n \)  Normal mode function
\( x \)  Longitudinal coordinate
\( \alpha \)  See Eq. (11.2.19)
\( \beta_n \)  Longitudinal wavenumber of the battery tab.  See Eq. (11.2.24)
\( \gamma \)  Wavenumber of a thin bar under longitudinal vibration
\( \lambda \)  Wavelength \( (\lambda = 2\pi/\gamma) \)
\( \rho \)  Mass density (2700 kg/m³ for aluminum and 8940 kg/m³ for copper)
\( \sigma_x \)  Axial stress
\( \sigma_Y \)  Yield strength \( (\sigma_Y = 55 \text{ MPa for aluminum and } \sigma_Y = 172 \text{ MPa for copper at } 25^\circ\text{C}) \)
\( \Omega \)  Sonotrode frequency [rad/s] \( (\Omega = 2\pi f, f = \text{sonotrode frequency [Hz]}) \)
\( \omega_n \)  Natural frequency [rad/s]
11.3 Dynamic Stress Analysis of Battery Tabs

One of the main issues arising in ultrasonic welding of battery tabs is to ensure consistent weld quality that meets design specifications such as the electrical conductivity and shear strength of the weld [7]. The quality of ultrasonic metal welds depends on a number of factors such as the weld configuration, mechanical and metallurgical properties of the weld parts, and weld process parameters – amplitude, clamping pressure, and welding energy [8, 9]. It is also important to carefully design the weld parts and tools to avoid resonance of any of the weld parts and tools to prevent failure during welding and post-welding. Through experimental and finite element analyses, Jagota and Dawson [3] showed that the bonding strength of thin-walled thermoplastic parts by ultrasonic welding is strongly influenced by the lateral vibration of the weld parts. The impact of waveform designs, by controlling the wavelength of the ultrasonic input, on vibration response reduction of weld parts for the battery welding system was studied by Lee et al. [4]. The effects of dynamic responses of weld parts on the sonotrode force required for welding in USMW was studied by Kang et al. [10, 11]. They showed that the sonotrode force is substantially affected by the weld spot location and also that the axial stresses of the battery tab during welding could exceed the material’s yield strength depending on the kinetic properties of the interface between the tab and battery cell pouch. However, in that study, the flexural vibration of the battery tab due to the transverse motion of the anvil during welding was not considered, which could cause large dynamic stresses near the weld spot under certain conditions. In our prior work [12], extensive experiments were conducted to gain understanding of the vibration characteristics of the ultrasonic welding process of battery tabs. Vibration response of the welder system including the anvil was measured using a high precision (resolution of 2 nm) single-point laser vibrometer at a sampling rate of 256 kHz. Experimental data consistently showed transverse (i.e., normal to the weld surface) vibrations of the anvil with amplitudes up to several microns. In particular, when the transfer stiffness of the anvil was deliberately reduced, the vibration amplitude of the anvil increased and some of the weld spots debonded or the tabs bulged near the weld spots. These experiments demonstrated that the transverse vibration of the anvil, even at the level of a few microns, could have a significant adverse effect on the weld quality of battery tabs.

While a significant amount of research work on USMW and its applications has been made, most efforts have been focused on the aspects of weld metallurgy and weldability of different materials. There is, however, only a limited amount of work to understand the overall dynamics of the ultrasonic welding system, particularly including the structural vibrations of weld parts and supporting structures (tools and fixtures). De Vries [2] summarized a number of studies related to USMW and presented a mechanics-based model to estimate the tangential forces during ultrasonic welding that act on the weld parts and interface, and correlate them to weld quality. Jagota and Dawson [3] presented experimental and finite element analyses showing that the bonding strength of thin-walled thermoplastic parts by ultrasonic welding was strongly influenced by the lateral vibration of the weld parts.

The present study is motivated by the observation of the sensitivity of weld quality, particularly the tab surface cracks and perforations, to the geometry, dimensions, and boundary conditions of tabs and busbar. These cracks and perforations are suspected to be low-cycle fatigue fractures [13, 14] induced by alternating axial stresses due to the vibration of the tab during welding. The objective of this study is to address the effects of geometry, dimensions, and boundary conditions of the battery tab (along with its material and mechanical properties) on the development of dynamic stresses induced by the longitudinal and flexural vibrations of the battery tab during ultrasonic welding. To this end, the battery tab is modeled as a continuous elastic beam extended in the direction parallel to the excitation direction of the sonotrode. In addition, the anvil vibration in the normal direction of the battery tab is considered as the excitation source to the flexural vibration of the tab during welding.

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2 The content presented in this section has previously appeared in reference [16].
3 Low-cycle fatigue fracture is associated with fatigue that occurs at lower than about $10^4$ to $10^5$ cycles [13].
11.3.1 Longitudinal and Flexural Vibrations of a Beam

The thickness of the battery tab is much smaller than its other dimensions. Therefore, the battery tab can be modeled as a thin beam extended parallel to the excitation direction of the sonotrode, and under longitudinal and flexural vibrations. A brief discussion of vibration theory [5, 15] related to the axial normal stress induced by the longitudinal and flexural vibrations of the battery tab is presented in this section.

Dynamic Stresses Due to Axial Boundary Excitation in a Thin Beam

Consider a thin beam, free at \( x = L \), under harmonic displacement excitation \( a_l \sin \Omega t \) at \( x = 0 \) in the axial direction as shown in Figure 11.15, where \( u \) denotes the axial displacement.

![Figure 11.15 Thin beam with coordinate x and longitudinal displacement u.](image)

The steady state longitudinal displacement of the beam due to the boundary excitation at \( x = 0 \) can be obtained by assuming a solution of the form

\[
u(x, t) = (C_1 \cos \lambda x + C_2 \sin \lambda x) \sin \Omega t \quad (11.3.1)
\]

where \( \lambda \) is the longitudinal wavenumber which is

\[
\lambda = \Omega/c_0 \quad c_0 = \sqrt{E/\rho} \quad (11.3.2)
\]

It can be seen that the velocity of the left end (i.e., \( x = 0 \)) of the beam must be the same as the excitation velocity at the steady state, thus

\[
\frac{\partial u(0, t)}{\partial t} = a_l \Omega \cos \Omega t \quad (11.3.3)
\]

In addition, the right end (i.e., \( x = L \)) of the beam must be stress-free as no constraint is imposed at that point, thus

\[
E \frac{\partial u(L, t)}{\partial t} = 0 \quad (11.3.4)
\]

Applying the above boundary conditions, the coefficients \( C_1 \) and \( C_2 \) can be determined, and therefore the steady state response of the beam due to the displacement excitation at \( x = 0 \) is found to be:

\[
u(x, t) = a_l (\cos \lambda x + \tan \lambda L \sin \lambda x) \sin \Omega t \quad (11.3.5)
\]

Notice that the response becomes unbounded (i.e. resonance occurs) when \( \cos \lambda L = 0 \) which is the characteristic equation for the longitudinal vibration of a fixed-free thin beam. Thus, one can find the longitudinal excitation frequencies that resonate the beam, which are:
\[ \Omega = \frac{(2n - 1)\pi}{2L} c_0 \quad n = 1, 2, 3, \ldots \]  

Once \( u(x, t) \) is known, the axial normal stress distribution along the thin beam can be determined by

\[ \sigma_l(x, t) = E \frac{\partial u(x, t)}{\partial x} = E a_l \lambda (\tan \lambda L \cos \lambda x - \sin \lambda x) \sin \Omega t \]  

In a similar manner, one can find the steady state stress distribution for other boundary conditions; for example, the axial normal stress distribution along a thin beam fixed at \( x = L \) is

\[ \sigma_l(x, t) = -E a_l \lambda (\sin \lambda x + \cot \lambda L \cos \lambda x) \sin (\Omega t + \phi_l) \]  

For this case, the response becomes unbounded when \( \sin \lambda L = 0 \), which is the frequency equation for the longitudinal vibration of a fixed-fixed thin beam. The resonant frequencies are:

\[ \Omega = \frac{n\pi}{L} c_0 \quad n = 1, 2, 3, \ldots \]  

<table>
<thead>
<tr>
<th>Boundary conditions</th>
<th>Aluminum</th>
<th>Copper</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( \rho = 2700 \text{ kg/m}^3 )</td>
<td>( \rho = 8940 \text{ kg/m}^3 )</td>
</tr>
<tr>
<td></td>
<td>( E = 70 \text{ GPa} )</td>
<td>( E = 110 \text{ GPa} )</td>
</tr>
<tr>
<td>Fixed-Free</td>
<td>( L = 127 \text{ mm} )</td>
<td>( L = 44 \text{ mm} )</td>
</tr>
<tr>
<td>Fixed-Fixed</td>
<td>( L = 254 \text{ mm} )</td>
<td>( L = 88 \text{ mm} )</td>
</tr>
</tbody>
</table>

Shown in Table 11.4 are the shortest beam length (i.e., \( n = 1 \)) corresponding to resonance at \( \Omega = 20 \text{ kHz} \) for two different materials, aluminum and copper. It can be seen that when the unconstrained tab is modeled as a thin beam, it will not resonate under either boundary conditions at \( x = L \) as its longitudinal length is much smaller than the resonant lengths.

**Dynamic Stresses Due to Transverse Boundary Excitation in a Thin Beam**

Consider a thin beam, free at \( x = L \), subjected to harmonic displacement excitation \( a_f \sin \Omega t \) at \( x = 0 \) in the transverse direction as shown in Figure 11.16, where \( w \) denotes the transverse displacement due to the flexural vibration of the beam.

\[ w(x, t) = (C_1 \cos \gamma x + C_2 \cosh \gamma x + C_3 \sin \gamma x + C_4 \sinh \gamma x) \sin \Omega t \]  

Figure 11.16  Thin beam with coordinate \( x \) and transverse displacement \( w \).

The steady state transverse displacement of the beam can be obtained by assuming a solution of the form
where $\gamma$ represents the flexural wavenumber which is defined as

$$\gamma^2 = \sqrt{A/l} \frac{\Omega}{c_0} \quad c_0 = \sqrt{E/\rho} \quad (11.3.11)$$

where $A = dh$ is the cross-sectional area and $l = dh^3/12$ the second area moment of inertia of the beam, respectively. The unknown coefficients $C_1$ through $C_4$ can be found by imposing the boundary conditions as follows. Due to the velocity continuity and zero slope at $x = 0$,

$$\frac{\partial w(0,t)}{\partial t} = a_f \Omega \cos \Omega t \quad \frac{\partial w(0,t)}{\partial x} = 0 \quad (11.3.12)$$

In addition, the bending moment $M$ and shear force $V$ due to bending must vanish at $x = L$, thus

$$M(L,t) = EI \frac{\partial^2 w(L,t)}{\partial x^2} \quad V(L,t) = -EI \frac{\partial^3 w(L,t)}{\partial x^3} = 0 \quad (11.3.13)$$

Upon application of the above boundary conditions, one can find that the steady state transverse response of the beam is to be

$$w(x,t) = a_f [\cosh \gamma x + C_A (\cos \gamma x - \cosh \gamma x) + C_B (\sin \gamma x - \sinh \gamma x)] \sin \Omega t \quad (11.3.14)$$

where

$$C_A = \frac{1}{2} \left( 1 - \frac{\sin \gamma L \sinh \gamma L}{1 + \cos \gamma L \cosh \gamma L} \right) \quad C_B = \frac{1}{2} \left( \frac{\cosh \gamma L \sin \gamma L + \cos \gamma L \sinh \gamma L}{1 + \cos \gamma L \cosh \gamma L} \right) \quad (11.3.15)$$

Notice that both $C_A$ and $C_B$ become unbounded, so does the transverse displacement $w$ of the beam, when

$$1 + \cos \gamma L \cosh \gamma L = 0 \quad (11.3.16)$$

which is the characteristic equation for a clamped-free thin beam under flexural vibration. The values of $\gamma L$ satisfying Eq. (11.3.16) are

$$\gamma_n L = \{1.875 \ 4.694 \ 7.855 \ 10.996 \ 14.137\} \quad \text{and} \quad \gamma_n \equiv (2n - 1)\pi/2L \quad \text{for} \ n > 5 \quad (11.3.17)$$

Therefore, combining with Eq. (11.3.14), the transverse excitation frequencies that resonate the clamped-free beam can be found by

$$\Omega = \gamma_n^2 \sqrt{EI/\rho A} \quad n = 1,2,3,\cdots \quad (11.3.18)$$

Once $w(x,t)$ is determined, the axial normal stress distribution (on the surface of the beam) at steady state due to bending caused by the flexural vibration of the beam can be found as

$$\sigma_f(x,t) = \frac{Ea_f h y^2}{2} [\cosh \gamma x - C_A (\cos \gamma x + \cosh \gamma x) - C_B (\sin \gamma x + \sinh \gamma x)] \sin \Omega t \quad (11.3.19)$$

In a similar manner, for a thin beam clamped at $x = L$, one can find that
\[
\sigma_f(x, t) = \frac{Ea_fh\gamma^2}{2} \left[ \cosh \gamma x - \mathcal{C}_A(\cos \gamma x + \cosh \gamma x) - \mathcal{C}_B(\sin \gamma x + \sinh \gamma x) \right] \sin \Omega t \quad (11.3.20)
\]

where

\[
\mathcal{C}_A = \frac{1}{2} \left( 1 + \frac{\sin \gamma L \sinh \gamma L}{1 - \cos \gamma L \cosh \gamma L} \right), \quad \mathcal{C}_B = \frac{1}{2} \left( \frac{\cosh \gamma L \sin \gamma L + \cos \gamma L \sinh \gamma L}{\cos \gamma L \cosh \gamma L - 1} \right) \quad (11.3.21)
\]

For this case, the response becomes unbounded when

\[
1 - \cos \gamma L \cosh \gamma L = 0 \quad (11.3.22)
\]

which is the frequency equation for the flexural vibration of a clamped-clamped thin beam. Accordingly, for this case, \(\gamma_n L\) corresponding to the resonant frequencies are

\[
\gamma_n L = \{4.730, 7.853, 10.996, 10.996, 17.279\} \text{ and } \gamma_n \equiv (2n + 1)\pi/2L \text{ for } n > 5 \quad (11.3.23)
\]

---

Shown in Figure 11.17 are the flexural natural frequency loci of a thin beam as a function of beam length \(L\) for two different materials, aluminum and copper. The cross-sectional dimensions of the beam are assumed to be \(d = 45\) mm and \(h = 0.2\) mm. The locus of curves shows the resonant vibration mode and the corresponding beam length at a given excitation frequency. For example, when \(\Omega = 20\) kHz, the aluminum clamped-free beam will resonate at its 1\(^{st}\) vibration mode if \(L\) is about 2.8 mm, while no resonance will occur if the beam is clamped at \(x = L\). However, it can be noticed that the \(n^{th}\) \((n \geq 1)\) resonant frequency of the clamped-clamped beam is almost the same\(^4\) as the \((n+1)^{th}\) resonant frequency of the clamped-free beam. This indicates that, except for the first mode of the clamped-free beam, resonance will occur at the same length under either boundary conditions at \(x = L\). That is, for example, for the

---

\(^4\) When higher order beam models such as the Timoshenko beam model that includes the effects of rotary inertia and shear deformation are used, the frequencies can be slightly different. The Timoshenko beam model is typically used for a beam whose slenderness ratio is less than 10.
aluminum beam with $L = 12$ mm and $\Omega = 20$ kHz, the 2nd vibration mode will be resonated if the beam is clamped at $x = L$, while the 3rd vibration mode will be resonated if the beam is free at $x = L$. It can be seen that the copper beam behaves in the same manner, with resonant beam lengths shorter than those of the aluminum beam.

11.3.2 Dynamic Stress in Battery Tab

As shown in Figure 11.18, the battery tab and other weld parts are clamped between the sonotrode tip and anvil. In this study, the tab is modeled as a thin beam under both longitudinal ($x$-direction) and flexural ($z$-direction) vibrations excited by the oscillatory motion of the weld spot element, based on the following observations and assumptions:

1) Only one tab is considered in the model; other tabs and the busbar (collectively called “other weld parts” in Figure 11.18) are considered as stationary rigid bodies. It should be noted that the dynamic coupling between the tab and other weld parts is negligible as the coupling is only through the weld spot area where the oscillating sonotrode tip is in contact with the tab.

2) The thickness of the tab is much smaller than the other dimensions of the tab, especially the tab length in the $x$-direction.

3) The anvil vibrates in the $z$-direction with a constant amplitude during welding as indicated in Figure 11.18, which causes the weld spot element to vibrate in phase and thus becomes the excitation source for the flexural vibration of the tab.

4) The weld spot element beneath the sonotrode tip is considered as a rigid body. It is assumed to oscillate in phase with the sonotrode tip and anvil.

5) The sonotrode tip oscillates in both $x$-direction and $z$-direction, where its motion in the $z$-direction is due to the rigid body coupling with the anvil via the weld spot element.

6) For modeling purpose, a term tab-end is defined for the cell-integrated tab. As indicated in Figure 11.18, the tab-end includes part of the tab extended from the bend line and some part of the battery cell pouch that surrounds the inserted tab. The kinetic properties of the tab-end are represented by equivalent stiffness and mass in both longitudinal and transverse directions. The effects of rotary inertia and rotational stiffness of the tab-end are not considered in this study.

7) The longitudinal and flexural vibrations of the tab are not coupled. This is a reasonable assumption as the vibration amplitudes are at micron levels and also the fundamental frequency of the longitudinal vibration of the tab is much higher than the excitation frequency (within the practical ranges of tab length) at which flexural vibration modes resonate.
The characteristics of the sonotrode excitation during welding are not altered by strain hardening of the materials at the weld interface. This is a reasonable assumption as the kinetic of the oscillating sonotrode is massive.

![Schematic of tab and its boundary conditions with equivalent mass and stiffness.]

The vibration characteristics of the tab are governed by the boundary conditions of the tab, then it can be seen from Figure 11.18 that the tab-end constitutes natural (kinetic) boundary conditions for the cell-integrated tab. During ultrasonic welding, part of the vibration energy injected by the oscillating sonotrode tip travels along the tab, through the tab-end, and then eventually dissipates in the battery cell pouch which contains viscoelastic materials. Hence, the kinetic properties of the tab-end become an important factor in determining both longitudinal and flexural vibration characteristics of the tab during ultrasonic welding. In this study, as shown in Figure 11.19, these kinetic properties of the tab-end are represented by the equivalent mass and equivalent stiffness. Since the tab-end undergoes both longitudinal and transverse motions, \( m_l \) and \( k_l \) represent the equivalent mass and stiffness, respectively, for the longitudinal motion; and \( m_f \) and \( k_f \) for the transverse motion of the tab-end. Due to complex geometry and material properties of the tab-end which consists of both parts of the battery tab and cell pouch, the determination of the equivalent mass and stiffness of the tab-end by analytical or numerical methods require extensive treatments including experiments. Experimental measurements of \( m_l \) and \( k_l \) of the tab-end for the longitudinal vibration are discussed in Section 11.2.3. It is, however, not a trivial task to experimentally measure \( m_f \) and \( k_f \) of the tab-end due to instrumentation challenges at present. Therefore, one objective of this study is to address the effects of \( m_f \) and \( k_f \) on the flexural vibration of the tab and resulting dynamic axial stresses through a parameter study by assuming a reasonable range of values for \( m_f \) and \( k_f \). For instance, considering the geometry of the tab-end, it is reasonable to assume that \( k_f < k_l \).

### Axial Normal Stresses in the Unconstrained Tab During Welding

The boundary conditions at \( x = L \) of the unconstrained battery tab are different from the ones of the cell-integrated tab used in the battery module, as previously discussed. When the unconstrained tab is clamped by the fixture during welding, the steady state axial stress distribution along the tab can be found by \( \sigma_l \) in Eq. (11.3.7) due to the longitudinal vibration and by \( \sigma_f \) in Eq. (11.3.19) due to bending caused by the flexural vibration of the tab. Figure 11.20, for example, shows the steady state \( \sigma_l \) and \( \sigma_f \) distributions along the aluminum unconstrained tab when the tab is free (Figure 11.20(a)) and clamped (Figure 11.20(b)) for \( L = 17 \) mm and \( \Omega = 20 \) kHz. Note that, as shown in Figure 11.21, when \( L = 17 \) mm, \( \Omega \) is very close to the 4th flexural vibration mode of the clamped tab and the 3rd flexural vibration mode of the free tab, therefore the overall magnitude of \( \sigma_f \) is larger than \( \sigma_l \) for both boundary conditions. However note that \( \sigma_l \) can be larger than \( \sigma_f \), especially at \( x = 0 \), when \( L \) is long enough so that \( \Omega \) is close to the fundamental frequency of the longitudinal vibration of the tab. These stress distribution curves also suggest that axial
stresses at certain points along the unconstrained tab, especially near the weld area (i.e., $x = 0$), can exceed the yield strength ($\sigma_Y = 55$ MPa for aluminum) of the tab during welding when resonance occurs. Note that the aluminum battery tab considered in this study is made of annealed Aluminum 1100 that is commercially pure aluminum (at least at 99% aluminum composition), soft, ductile, low strength, and has excellent welding characteristics.

Figure 11.20  Axial stress distribution along the aluminum unconstrained tab when $\Omega = 20$ kHz and $L = 17$ mm for the (a) free boundary and (b) clamped boundary at $x = L$.

Figure 11.21  $\sigma_0$ of the unconstrained tab as a function of tab length $L$ for (a) aluminum and (b) copper tabs when $\Omega = 20$ kHz.

As previously mentioned, cracks occur near the weld area of the unconstrained tab when the lower end of the tab is not clamped by the fixture during welding. These cracks are believed to be fatigue fractures as a result of large vibration-induced axial stresses near the weld area. In order to evaluate the maximum stress near the weld area, denote $\sigma_0$ as the total axial stress at $x = 0$. Note that $\sigma_0$ is defined as the superposition of the individual stress magnitudes (i.e., $\sigma_0 = |\sigma_f(0)| + |\sigma_r(0)|$) caused by the two uncoupled vibrations by assuming that $\sigma_f(0)$ and $\sigma_r(0)$ are in phase, thus $\sigma_0$ can be considered as the possible maximum axial stress for the worst case of scenario from the tab design point of view. Shown in
Figure 11.21 is $\sigma_0$ as a function of tab length $L$ for the unconstrained tab with two different boundary conditions at $x = L$. Notable findings are as follows.

- $\sigma_0$ of the unconstrained tab depends on the boundary conditions of the tab at $x = L$. $\sigma_0$ is larger when the tab end is clamped at $x = L$ up to $L$ is about 30 mm for the aluminum tab and up to 20 mm for the copper tab. The gradual increase (decrease) of the lower bound of $\sigma_0$ is due to the increase (decrease) of the stress due to the longitudinal vibration of the tab.

- $\sigma_0$ is large, beyond the yield strength, when the tab resonates at one of its flexural vibration modes. For example, as shown in Figure 11.21(a), $\sigma_0$ of the aluminum tab is higher than its $\sigma_Y$ when $L = 31$ mm, however it is significantly lower than $\sigma_Y$ when $L = 30$ mm or $L = 29$ mm. Note that the aluminum tab with free end at $x = L$ resonates at its 7th flexural vibration mode when $L = 31.2$ mm. This confirms the experimental observations of fatigue cracks occurring near the weld area when $L = 31$ mm.

![Figure 11.21](image1)

Figure 11.22 shows $\sigma_0$ of the cell-integrated aluminum tab as a function of tab length $L$. (a) Effect of $k_f$ when $m_f = 0$, (b) effect of $m_f$ when $k_f = 0$, (c) effect of $m_f$ when $k_f = 25$ kN, and (d) effect of $m_f$ when $k_f = 50$ kN.
Axial Normal Stresses in the Cell-integrated Tab During Welding

Figure 11.22 shows $\sigma_0$ as a function of tab length $L$ for the cell-integrated aluminum tab with different values of $k_f$ and $m_f$ of the tab-end, where $k_l = 100$ kN/m and $m_l = 0.063$ grams [10] are assumed. Notable findings are as follows.

- As shown in Figure 11.22(a), $\sigma_0$ of the cell-integrated tab behaves in the same manner as the case for the tab-end free from constraints when $m_f = 0$; that is $k_f$ has an insignificant effect on changing $\sigma_0$ unless $k_f$ is very large. It is realistic to assume that $k_f < k_l$.

- The effect of $m_f$ on $\sigma_0$ is more pronounced than $k_f$ as shown in Figure 11.22(b). Increasing $m_f$ lowers the natural frequencies of the tab, and thus shortens the tab length $L$ that resonates at 20 kHz. For example, if $L$ is chosen to be between 8mm to 10 mm, $\sigma_0$ may increase or decrease with increasing $m_f$ depending on $L$; however, it can be seen that $\sigma_0$ is still lower than $\sigma_Y$. Noting that $m_l$ ranges from 0.002 to 0.01 grams (Section 11.2.3) for the aluminum tab, hence the range of values for $m_f$ used in this analysis can be considered to be conservative.

- The effect of $m_f$ on $\sigma_0$ with various non-zero values of $k_f$ has been examined. Shown in Figure 11.22(c) and Figure 11.22(d) are two representative cases with $k_f = 25$ kN/m and $k_f = 50$ kN/m, respectively. When the two cases are compared, it can be seen that the effect of $m_f$ remains almost unchanged. This is also true even for $k_f$ larger than $k_l$ as long as $k_f$ is not unrealistically large.

- It can be clearly seen that the weld location has a critical impact on $\sigma_0$.

Shown in Figure 11.23 is $\sigma_0$ as a function of tab length $L$ for the cell-integrated copper tab with different values of $k_f$ and $m_f$ of the tab-end, where $k_l = 150$ kN/m and $m_l = 0.01$ grams (see Section 11.2.3) are assumed. The overall behavior of $\sigma_0$ as well as the effects of $k_f$ and $m_f$ are similar to the aluminum tab. However, for this case, notice that the upper bound of the current practice range of $L$ is close to one of the resonant lengths. It should be also noted that high axial normal stresses in the tab during welding increase the sonotrode force required for welding.

![Graph](image_url)

Figure 11.23  $\sigma_0$ of the cell-integrated tab as a function of tab length $L$. (a) Effect of $k_f$ when $m_f = 0$ and (b) effect of $m_f$ when $k_f = 0$. 
11.3.3 Summary and Conclusions

The distribution of axial normal stresses in the battery tab during ultrasonic welding is studied in this work by applying an analytical mechanistic model. The battery tab is modeled as a thin beam under longitudinal and flexural vibrations due to ultrasonic excitation from the sonotrode. The main focus of this study is to assess the effects of the elastic vibration of the battery tab on the stress development near the weld spot area during welding. This study serves as the foundation for a scientific understanding of battery tab dynamics during ultrasonic welding and its effect on weld quality, and thus provides a guideline for design and welding of battery tabs. It is found that High stresses can develop when the ultrasonic welding frequency (nominally 20 kHz) is close to one of the tab’s natural frequencies. The natural frequencies of the tab depend on the length between the weld spot and tab-end (interface between the battery tab and cell pouch), boundary conditions of the tab-end, cross-sectional area, and material of the tab. Therefore, it is important to design the weld position on the tab in the ultrasonic welding process such that the tab’s natural frequencies stay away from 20 kHz as much as possible to minimize the tab stresses. Stresses near the weld area are mainly due to the flexural vibration of the tab during welding, which could exceed the tab material’s yield strength and cause fatigue fracture. These stresses can be significantly reduced by reducing the anvil vibration. Stresses due to the longitudinal vibration of the tab during welding are insignificant.

Nomenclature

\( A \) Cross-sectional area of battery tab (45 mm × 0.2 mm)
\( a_f \) Sonotrode amplitude in the transverse direction (5 \( \mu \)m is used for calculations)
\( a_l \) Sonotrode amplitude in the longitudinal direction (10 \( \mu \)m is used for calculations)
\( c_0 \) Phase velocity \( (c_0 = \sqrt{E/\rho}) \)
\( E \) Young’s modulus (70 GPa for aluminum and 110 GPa for copper)
\( k_f \) Equivalent stiffness of the tab-end in transverse motion (see Figure 11.19)
\( k_l \) Equivalent stiffness of the tab-end in longitudinal motion (see Figure 11.19)
\( L \) Tab length (see Figure 11.19)
\( m_f \) Equivalent mass of the tab-end in transverse motion (see Figure 11.19)
\( m_l \) Equivalent mass of the tab-end in longitudinal motion (see Figure 11.19)
\( t \) Time
\( u \) Longitudinal displacement of the tab
\( U_n \) Normal mode function
\( w \) transverse displacement of the tab
\( x \) Longitudinal coordinate
\( \gamma \) Wavenumber of a thin beam under longitudinal vibration
\( \lambda \) Wavelength \( (\lambda = 2\pi/\gamma) \)
\( \rho \) Mass density (2700 kg/m\(^3\) for aluminum and 8940 kg/m\(^3\) for copper)
\( \sigma_0 \) Axial normal stress at \( x = 0 \)
\( \sigma_f \) Axial normal stress due to flexural vibration
\( \sigma_l \) Axial normal stress due to longitudinal vibration
\( \sigma_Y \) Yield strength \( (\sigma_Y = 55 \text{ MPa for aluminum and } \sigma_Y = 172 \text{ MPa for copper at } 25^\circ C) \)
\( \Omega \) Sonotrode frequency [rad/s] \( (\Omega = 2\pi f, f = \text{ sonotrode frequency [Hz]}) \)
\( \omega_n \) Natural frequency [rad/s]
11.4 Vibrational Energy Loss Analysis in Battery Tab Ultrasonic Welding

There are three quality indices in USMW; i.e., bonding effectiveness between weld parts [9], cracks/perforation on the weld surfaces [9], and bulging/distortion of the weld parts [10, 16]. The weld quality depends on a number of controllable factors such as mechanical and metallurgical properties of the weld parts, weld part geometry and dimensions, weld configuration, weld tool (e.g., the sonotrode and anvil) design, and welding process parameters — sonotrode amplitude, clamping pressure, and welding energy. In this regard, a significant amount of research has been made; for example, work by Lee et al. [9], Lee et al. [4], Li et al. [17], and Zhao et al. [1].

The weld quality also depends a number of uncontrollable factors such as weld tool alignment, tool wear, work part surface variations and contaminations, and, as a unique characteristics in ultrasonic welding, the dynamics of the ultrasonic welding system, particularly the structural vibrations of weld parts and supporting structures (tools and fixtures). By experimental and finite element analyses, Jagota and Dawson [3] showed that the bonding strength of thin-walled thermoplastic parts by ultrasonic welding is strongly influenced by the lateral vibration of the weld parts. The impact of waveform designs, by controlling the wavelength of the ultrasonic input, on vibration response reduction of weld parts for the battery welding system was studied by Lee at al. [4]. The effect of dynamic responses of weld parts on the sonotrode force required for welding in USMW was studied by Kang et al. [10]. In our prior work [12], extensive experiments were also conducted to gain understanding of the vibration characteristics of the ultrasonic welding process of battery tabs. The experiments were designed to weld three tabs (which were not attached to battery cell pouches; the tab ends were clamped to a fixture) to the bus-bar (Cu coupon). Vibration response of the welder system was measured by a high precision (resolution of 2 nm) Polytec single-point laser vibrometer. The vibrometer was synchronized with the welder system and a National Instruments DAQ system. The frequency of the sonotrode oscillation during welding was about 20 kHz and data were sampled at 200 kHz. Data were post-processed using the laser vibrometer software and MATLAB to obtain the response, FFT and other critical characteristics. In a set of experiments, the stiffness of the anvil was deliberately reduced, and Polytec laser vibrometer measured several microns more anvil vibration (almost doubled), adversely affecting the weld quality. In particular, some of the weld spots could debond and the tabs could also bulge. This experimental study demonstrated that undesirable anvil vibrations, even at the level of a few microns, could have significant adverse effects on the weld quality. This study also clearly indicated that although the sole vibration source is from the sonotrode, anvil does vibrate due to the coupling effect since the sonotrode applies high clamping force onto the work parts against the anvil.

Since the more system vibrates, the more energy loss is. Thus, the objective of this work is to study the observed phenomena from vibrational energy point of view. We believe it is very important to minimize vibrational energy loss by carefully designing the weld parts and tools, especially to avoid resonance of any of the weld parts and tools as considerable amounts of energy are consumed through resonant vibrations. Therefore, the present work examines the vibration energy loss of the bus-bar coupon due to the longitudinal and flexural vibrations of the overhang (upper part of the bus-bar extended from the anvil) during ultrasonic welding and assess the effects of this energy loss on the weld strength of battery tabs. The overhang of the bus-bar is modeled as a thin beam extended in the direction parallel to the excitation direction of the sonotrode. Sections 11.4.1 and 11.4.2 present the principle of work and energy loss for forced longitudinal and flexural vibrations of thin beams. In Section 11.4.3, vibrational energy loss associated with the vibration of the overhang is discussed. Section 11.4.4 describes the experimental estimation of material damping of the bus-bar coupon. Summary and conclusions are presented in Section 11.4.5.

11.4.1 Energy Dissipation by Material Damping in Longitudinal Vibration

In ultrasonic welding of battery tabs, the thickness of the bus-bar coupon is much smaller than its other dimensions and the loading conditions are symmetric. Thus, in the present study, the overhang of the bus-bar coupon is modeled as a thin, internally damped beam extended parallel to the direction of the excitation

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5 The content presented in this section has previously appeared in reference [11].
of the sonotrode. An overview of the theory needed to calculate the vibrational energy loss of the bus-bar coupon is presented in this section. Using the modal analysis approach, the energy loss is expressed as a sum of the modal energies. The frequency characteristics of modal energy loss, which is essential to understand the vibrational energy loss in the bus-bar coupon during ultrasonic welding, is presented using a single degree of freedom system in Chapter 10.

Consider a thin, infinitely long, undamped, straight bar with a uniform cross-section subjected to an arbitrarily distributed axial body force \( P(x, t) \) (measured as a force per unit length) as shown in Figure 11.24(a). The equation governing the longitudinal vibration of the bar can be found as [5]

\[
\rho A \frac{\partial^2 u}{\partial t^2} = EA \frac{\partial^2 u}{\partial x^2} + P(x, t)
\]

where \( u = u(x, t) \) denotes the axial displacement of a cross-section, \( x \) the spatial coordinate, \( t \) the time, \( E \) the Young’s modulus, \( A \) the cross-sectional area, and \( \rho \) the mass density of the bar. In the absence of the body force, Eq. (11.4.1) reduces to the classical wave equation:

\[
\frac{1}{c_0^2} \frac{\partial^2 u}{\partial t^2} = \frac{\partial^2 u}{\partial x^2} \quad c_0 = \sqrt{\frac{E}{\rho}}
\]

where \( c_0 \) is known as the phase velocity (or bar velocity) at which longitudinal waves propagate. Typical phase velocities in most metals are quite high compared to the velocity of sound in air of 340 m/s; for example, copper has \( c_0 = 3,508 \text{ m/s} \). The general solution to Eq. (11.4.1) can be found by assuming that

\[
u(x, t) = U(x)G(t)
\]

where

\[
U(x) = C_1 \cos \gamma x + C_2 \sin \gamma x
\]

\[
G(t) = D_1 \cos \omega t + D_2 \sin \omega t
\]

where the radial frequency \( \omega \), wavenumber \( \gamma \), and wavelength \( \lambda \) (the distance between two successive points of constant phase) are related by

\[
\omega = c_0 \gamma = \frac{2\pi c_0}{\lambda}
\]

The arbitrary constants in Eqs. (11.4.4) and (11.4.5) depend on the boundary conditions and initial conditions. For example, consider a bar fixed at one end \( (x = 0) \) and free at the other end \( (x = L) \). The
fixed boundary condition at $x = 0$ implies that the displacement at the end must be zero, therefore

$$u(0, t) = U(0)G(t) = C_1 G(t) = 0 \quad (11.4.7)$$

Since $G(t) \neq 0$ for all time, Eq. (11.4.7) dictates that $C_1 = 0$. In addition, the free boundary condition at $x = L$ requires that the stress at the end must vanish; i.e.,

$$EA \frac{\partial u(L, t)}{\partial x} = EA \frac{dU(L)}{dx} G(t) = EA \gamma C_2 \cos \gamma L G(t) = 0 \quad (11.4.8)$$

Since $C_2 \neq 0$, $\gamma \neq 0$, and $G(t) \neq 0$, it can be found that

$$\cos \gamma L = 0 \quad (11.4.9)$$

which is the frequency equation for the fixed-free bar under longitudinal vibration. Eq. (11.4.9) is satisfied only when

$$\gamma_n = \frac{(2n - 1)\pi}{2L} \quad n = 1, 2, 3, \ldots \quad (11.4.10)$$

Thus, the natural frequencies of the system are

$$\omega_n = \frac{(2n - 1)\pi}{2L} c_0 \quad n = 1, 2, 3, \ldots \quad (11.4.11)$$

These represent the discrete frequencies at which the system is capable of undergoing resonance. For a given $n$, the vibrational pattern (called the $n^{th}$ normal mode or mode shape) of the bar is described by

$$U_n(x) = \sin \gamma_n x \quad n = 1, 2, 3, \ldots \quad (11.4.12)$$

Combining the time and spatial dependence for a given $n$, the assumed solution in Eq. (11.4.3) becomes

$$u_n(x, t) = (D_{1n} \cos \omega_n t + D_{2n} \sin \omega_n x) \sin \gamma_n x \quad (11.4.13)$$

The general solution is then obtained by superposing all particular solutions as

$$u(x, t) = \sum_{n=1}^{N} u_n(x, t) = \sum_{n=1}^{N} (D_{1n} \cos \omega_n t + D_{2n} \sin \omega_n x) \sin \gamma_n x \quad (11.4.14)$$

where the coefficients $D_{1n}$ and $D_{2n}$ are to be determined by applying the initial conditions of the bar.

Now, consider a fixed-free bar of length $L$, Figure 11.24(b), with Kelvin-Voigt material damping (internal damping) and subjected to an axially distributed force $P(x, t)$. The corresponding equation of motion governing the longitudinal vibration takes the following form

$$\rho A \frac{\partial^2 u}{\partial t^2} = c \frac{\partial^3 u}{\partial x^3 \partial t} + EA \frac{\partial^2 u}{\partial x^2} + P(x, t) \quad (11.4.15)$$

Note that Kelvin-Voigt damping is a viscoelastic model commonly used for metals with relatively small damping. In this model, the stress $\sigma$, strain $\epsilon$, and its rate of change with respect to time are related by:
\[ \sigma = E\varepsilon + c \frac{d\varepsilon}{dt} \quad (11.4.16) \]

where \( c \) represents the viscosity of the material. The solution to Eq. (11.4.15) can be written in terms of the normal modes associated with undamped system as

\[ u(x, t) = \sum_{n=1}^{N} q_n(t) U_n(x) \quad (11.4.17) \]

where \( U_n(x) \) is the \( n \)th normal mode function for the fixed-free bar as shown in Eq. (11.4.12) and \( q_n(t) \), referred to as time-dependent generalized coordinates or modal coordinates \([15]\) in modal analysis of discrete systems, satisfies the following equation:

\[ \ddot{q}_n + 2\zeta_n \omega_n \dot{q}_n + \omega_n^2 q_n = Q_n(t) \quad (11.4.18) \]

\[ Q_n(t) = \frac{1}{\rho A \alpha_n} \int_0^L P(x, t) U_n(x) dx \quad \zeta_n = \frac{c\alpha_n}{2EA} \quad \alpha_n = \int_0^L U_n^2(x) dx \quad (11.4.19) \]

Note that the basic problem of Eq. (11.4.18) is the vibration of a single degree of freedom system. \( Q_n(t) \) is the generalized force or modal force in modal analysis of discrete systems associated with \( q_n(t) \) and \( \zeta_n \) is the modal damping ratio. If \( P(x, t) = P_0 \sin \Omega t \), i.e., a uniformly distributed harmonic body force, one can find that

\[ q_n(t) = \frac{P_0}{\rho A \alpha_n \omega_d} \int_0^t \sin \Omega \tau e^{-\zeta_n \omega_d (t-\tau)} \sin \omega_d (t - \tau) d\tau \quad \omega_d = \omega_n \sqrt{1 - \zeta_n^2} \quad (11.4.20) \]

Once \( u(x, t) \) in Eq. (11.4.17) is found, the mechanical work \( W \) performed by the axial force on the harmonic motion of the bar over a time span of \( t_0 \) can be determined in a straight manner as follows

\[ W = \int_0^t \int_0^{t_0} P(x, \tau) \dot{u}(x, \tau) d\tau \, dx \]

\[ = \int_0^{t_0} \int_0^L P(x, \tau) \left( \sum_{n=1}^{N} \dot{q}_n(\tau) U_n(x) \right) d\tau \, dx \quad (11.4.21) \]

\[ = \rho A \sum_{n=1}^{N} \alpha_n \int_0^{t_0} \dot{q}_n(\tau) Q_n(\tau) \, d\tau \]

In addition, the kinetic energy \( T \) at \( t = t_0 \) is

\[ T = \frac{1}{2} \rho A \int_0^L \left( \frac{\partial u}{\partial t} \right)^2 dx = \frac{1}{2} \rho A \sum_{n=1}^{N} \alpha_n \dot{q}_n^2(t_0) \quad (11.4.22) \]

and the elastic potential energy \( V \) at \( t = t_0 \) is
\[ V = \frac{1}{2} EA \int_0^L \left( \frac{\partial u}{\partial x} \right)^2 \, dx = \frac{1}{2} EA \sum_{n=1}^N q_n^2(t_0) \int_0^L \left( \frac{dU_n}{dx} \right)^2 \, dx \]  

(11.4.23)

Lastly, the energy \( W_d \) dissipated by material damping over a time span \( t_0 \) can be found as

\[
W_d = -\int_0^{t_0} \int_0^L c \left( \frac{\partial^3 u}{\partial x^2 \partial t} \right) \frac{\partial u}{\partial \tau} \, d\tau \, dx
= -c \int_0^{t_0} \int_0^L \left( \sum_{n=1}^N \ddot{q}_n(\tau)U_n^u(\chi) \right) \left( \sum_{n=1}^N \ddot{q}_n(\tau)U_n(\chi) \right) \, dx \, d\tau
= 2\rho A \sum_{n=1}^N \alpha_n \zeta_n \omega_n \int_0^{t_0} \dot{q}_n^2(\tau) \, d\tau
\]

(11.4.24)

It should be noted that the work done \( W \) by the harmonic body force \( P(x,t) \) in Eq. (11.4.21) and the energy dissipated by damping \( W_d \) in Eq. (11.4.24) include both the transient and steady-state responses as the solution in Eq. (11.4.17) includes both response solutions. Moreover, energies in Eqs. (11.4.21) to (11.4.24) are expressed in terms of the modal energies, i.e., energies associated with the modal coordinates, whose fundamental characteristics in the frequency domain have been described in Chapter 10.

### 11.4.2 Energy Dissipation by Material Damping in Flexural Vibration

Consider a thin bar, clamped at \( x = 0 \) and free at \( x = L \), under flexural (transverse) vibration due to a body force \( F(x,t) \) distributed over the span, as shown in Figure 11.25. Denoting \( x \) as the spatial variable and \( t \) the time variable, the equation governing the transverse displacement \( w(x,t) \) under the flexural vibration of the bar is [5]

\[
\rho A \frac{\partial^2 w}{\partial t^2} + EI \frac{\partial^4 u}{\partial x^4} = F(x,t)
\]

(11.4.25)

where \( I \) is the second area moment of inertia of the bar; i.e., \( I = dh^3/12 \). In free vibration, the general solution of Eq. (11.4.25) can be found by assuming that

\[
w(x,t) = W(x)G(t)
\]

(11.4.26)

Application of the above assumed solution to Eq. (11.4.25) leads to

\[
W(x) = C_1 \cos \gamma x + C_2 \cosh \gamma x + C_3 \sin \gamma x + C_4 \sinh \gamma x
\]

(11.4.27)
\[ G(t) = D_1 \cos \omega t + D_2 \sin \omega t \]  
(11.4.28)

where, the radial frequency \( \omega \) and wavenumber \( \gamma \) are related by

\[ \omega^2 = \frac{EI}{\rho A} \gamma^4 \]  
(11.4.29)

The arbitrary constants in Eqs. (11.4.27) and (11.4.28) depend on the boundary conditions and initial conditions. For clamped boundary conditions at \( x = 0 \), the displacement and slope must be zero. Hence,

\[ w(0, t) = (C_1 + C_2)G(t) = 0 \]  
(11.4.30)

\[ \frac{\partial w(0, t)}{\partial x} = \gamma(C_3 + C_4)G(t) = 0 \]  
(11.4.31)

In addition, the free boundary conditions at \( x = L \) require that the bending moment and shear force at the end must vanish; i.e.,

\[ EI \frac{\partial^2 w(0, t)}{\partial x^2} = -EI \gamma^2(C_1 \cos \gamma L - C_2 \cosh \gamma L + C_3 \sin \gamma L - C_4 \sinh \gamma L)G(t) \]  
(11.4.32)

\[ EI \frac{\partial^3 w(0, t)}{\partial x^3} = -EI \gamma^3(C_1 \sin \gamma L + C_2 \sinh \gamma L - C_3 \cos \gamma L + C_4 \cosh \gamma L)G(t) \]  
(11.4.33)

Since \( \gamma \neq 0 \) and \( G(t) \neq 0 \), in order for Eqs. (11.4.30) through (11.4.33) to have a nontrivial solution, wavenumber \( \gamma \) must satisfy the following equation

\[ \cos \gamma L \cosh \gamma L + 1 = 0 \]  
(11.4.34)

which is the frequency equation for the clamped-free thin bar under flexural vibration. The lowest five values of \( \gamma L \) satisfying Eq. (11.4.34) are

\[ \gamma_n L = \{1.875 \ 4.694 \ 7.855 \ 10.996 \ 14.137\} \quad n = 1, 2, \ldots, 5 \]  
(11.4.35)

and \( \gamma_n L \equiv (2n - 1)\pi/2 \) for \( n > 5 \). Accordingly, the natural frequency can be found from

\[ \omega_n = \frac{\gamma_n^2}{L^2} \sqrt{\frac{EI}{\rho A}} \quad n = 1, 2, 3, \ldots \]  
(11.4.36)

These represent the discrete frequencies at which the system undergoes harmonic motion. For a given \( n \), the \( n^{th} \) normal mode (modeshape function) of the bar is

\[ W_n(x) = \cosh \gamma_n x - \cos \gamma_n x - \frac{\sinh \gamma_n L - \sin \gamma_n L}{\cosh \gamma_n L + \cos \gamma_n L} \frac{\sin \gamma_n x - \sin \gamma_n L}{\cosh \gamma_n L + \cos \gamma_n L} \quad n = 1, 2, 3, \ldots \]  
(11.4.37)

Combining the time and spatial dependence for a given \( n \), the assumed solution in Eq. (11.4.26) becomes
\[ w_n(x, t) = (D_{1n} \cos \omega_n t + D_{2n} \sin \omega_n t)W_n(x) \]  
(11.4.38)

The general solution is then obtained by superposing all particular solutions as

\[ w(x, t) = \sum_{n=1}^{N} w_n(x, t) = \sum_{n=1}^{N} (D_{1n} \cos \omega_n t + D_{2n} \sin \omega_n t) W_n(x) \]  
(11.4.39)

where the coefficients \( D_{1n} \) and \( D_{2n} \) are to be determined by applying the initial conditions of the thin bar.

Now, consider a clamped-free bar of length \( L \) with Kelvin-Voigt material damping and subjected to a distributed body force \( F(x, t) \). The corresponding equation of motion governing the flexural vibration is

\[ \rho A \frac{\partial^2 w}{\partial t^2} + c \frac{\partial^4 w}{\partial t^2} + EI \frac{\partial^4 u}{\partial x^4} = F(x, t) \]  
(11.4.40)

In the same as determining the solution for the longitudinal vibration problem in Section 11.4.1, the solution to Eq. (11.4.40) can be written in terms of the normal modes associated with undamped system as

\[ w(x, t) = \sum_{n=1}^{N} q_n(t)W_n(x) \]  
(11.4.41)

where \( W_n(x) \) is the normal mode function for the clamped-free bar as shown in Eq. (11.4.37) and the \( n^{th} \) time-dependent generalized coordinate \( q_n(t) \) satisfies the following equations

\[ \ddot{q}_n + 2\zeta_n \omega_n \dot{q}_n + \omega_n^2 q_n = Q_n(t) \quad n = 1, 2, 3, \ldots \]  
(11.4.42)

\[ Q_n(t) = \frac{1}{\rho A \alpha_n} \int_0^L F(x, t)W_n(x)dx \]

\[ \zeta_n = \frac{c \omega_n}{2EI} \quad \alpha_n = \int_0^L W_n^2(x)dx \]  
(11.4.43)

Again, the basic problem of Eq. (11.4.42) is the vibration of a single degree of freedom system. \( Q_n(t) \) is the generalized force associated with \( q_n(t) \) and \( \zeta_n \) is the modal damping ratio. If \( F(x, t) = F_0 \sin \Omega t \), i.e., a uniformly distributed harmonic body force, it can be found that

\[ q_n(t) = \frac{F_0}{\rho A \alpha_n \omega_d} \int_0^L W_n(x)dx \int_0^t \sin \Omega t e^{-\zeta_n \omega_d(t-\tau)} \sin \omega_d(t-\tau)d\tau \quad \omega_d = \omega_n \sqrt{1 - \zeta_n^2} \]  
(11.4.44)

Once \( w(x, t) \) in Eq. (11.4.41) is obtained in terms of the normal modes, the mechanical work \( W \) performed by \( F(x, t) \) on the harmonic flexural motion of the clamped-free thin bar over a time span of \( t_0 \) can be determined in a similar manner as for the previous longitudinal vibration case, which is

\[ W = \int_0^{t_0} \int_0^L F(x, \tau) \dot{w}(x, \tau) d\tau dx = \int_0^{t_0} \int_0^L F(x, \tau) \left( \sum_{n=1}^{N} \dot{q}_n(\tau)W_n(x) \right) dx d\tau \]  
(11.4.45)

\[ = \rho A \sum_{n=1}^{N} \alpha_n \int_0^{t_0} \dot{q}_n(\tau)Q_n(\tau)d\tau \]
The kinetic energy at $t = t_0$ becomes

$$T = \frac{1}{2} \rho A \int_0^L \left( \frac{\partial w}{\partial t} \right)^2 \, dx = \frac{1}{2} \rho A \sum_{n=1}^N \alpha_n \dot{q}_n^2(t_0)$$  \hspace{1cm} (11.4.46)

and the elastic potential energy at $t = t_0$ is

$$V = \frac{1}{2} EI \int_0^L \left( \frac{\partial^2 w}{\partial x^2} \right)^2 \, dx = \frac{1}{2} EI \sum_{n=1}^N q_n^2(t_0) \int_0^L \left( \frac{d^2 W_n}{dx^2} \right)^2 \, dx$$  \hspace{1cm} (11.4.47)

The energy $W_d$ dissipated by material damping over a time span of $t_0$ for the clamped-free thin bar under flexural vibration is

$$W_d = \int_0^t \int_0^L \frac{c}{\partial x^4} \left( \frac{\partial w}{\partial x} \right) \left( \frac{d^4 W_n}{dx^2} \right) \, dx \, d\tau$$

$$= c \int_0^t \int_0^L \left( \sum_{n=1}^N \dot{q}_n(\tau) \frac{d^4 W_n(x)}{dx^4} \right) \left( \sum_{n=1}^N q_n(\tau) W_n(x) \right) \, dx \, d\tau$$  \hspace{1cm} (11.4.48)

Again, it should be noted that the work done $W$ by the harmonic body force $F(x, t)$ in Eq. (11.4.45) and the energy dissipated by damping $W_d$ in Eq. (11.4.48) include both the transient and steady-state responses as the solution in Eq. (11.4.41) is not limited to the steady-state response.

### 11.4.3 Vibrational Energy Loss of the Overhang

Figure 11.26 shows a schematic of the current ultrasonic welding setup for battery tabs and bus-bar (U/W-channel), where the battery tabs and bus-bar are clamped between the sonotrode tips and the anvil. The weld quality of battery tabs, measured by pull/peel tensile forces, perforation, and bulging, is affected by the dimensions of the bus-bar, especially by the overhang length ($L$) of the bus-bar. Longitudinal and transverse vibrations (in the $x$ and $y$ directions, respectively) of the anvil, of the order of several microns, can cause the overhang to vibrate in those respective directions. With different $L$, these vibrations are suspected to cause the differences in the weld quality. In this study, the vibrational energy loss associated with the overhang is estimated and correlated to the weld strength to assess the effects of the overhang length on the weld quality. We model the overhang of the bus-bar as a thin bar vibrating in both the longitudinal and transverse directions under body force excitation due to the anvil vibrations.

The following observations and assumptions are imposed in our model:

1) The vibrations of the overhang of the bus-bar are caused primarily by the vibrations of the anvil, i.e., the anvil can be considered a moving support for the overhang of the bus-bar. For this case, the excitation force at the anvil and overhang interface can be transformed into an inertial body force uniformly distributed over the span of the overhang, especially when the excitation frequency does not resonate a high vibration mode of the overhang [18]. Under the current design of the bus-bar and welding setup, $L \leq 10$ mm, and for such a short span length, the dominant vibration mode of the overhang is the fundamental (first) mode for both longitudinal and flexural vibrations. For example, when $L = 10$ mm and $t = 0.9$ mm (thickness), $\omega_1 = 5.1$ kHz and $\omega_2 = 32$ kHz for the flexural vibration and $\omega_1 = 87.7$ kHz and $\omega_2 = 263$ kHz for the longitudinal vibration.
2) The thickness of the overhang is much smaller than the other dimensions of the overhang.

3) The longitudinal and flexural vibrations of the overhang are uncoupled. This is a reasonable assumption as the natural frequencies of the longitudinal vibration are much higher than those of the flexural vibration. In addition, vibration amplitudes of the overhang can be considered very small so that linear elasticity theory is applicable.

4) The following values of the physical parameters are used in the numerical simulation results:

- Material properties: \( \rho = 8,940 \text{ kg/m}^3 \) and \( E = 110 \) for copper
- Dimensions: \( A = dh, d = 45 \text{ mm}, \) and \( h = 0.4 - 0.9 \text{ mm} \)
- Welding parameters: \( a_l = 10 \mu\text{m}, a_f = 5 \mu\text{m}, \) and \( \Omega = 20 \text{ kHz} \)
- \( P_0 = \rho A a_l \Omega^2 = 25.4 - 57.2 \text{ kN/m} \)
- \( F_0 = \rho A a_f \Omega^2 = 12.7 - 28.6 \text{ kN/m} \)
- Modal damping ratio: \( \zeta_n = 0.001, 0.003, 0.01, \) and \( 0.02 \) for both longitudinal and flexural modes. A series of experiments was performed to estimate the damping ratio of the bus-bar coupon in ultrasonic welding (see the Appendix at the end of current chapter). The damping ratios chosen in the numerical simulations are based on the range of estimated values of \( \zeta \) from the experiments.

Energy Loss Due to Longitudinal Vibration

The energy dissipated by material damping in the longitudinal vibration of the overhang is estimated. From Section 11.4.1, the mechanical work \( W \) performed by the axially distributed body force \( P(x, t) = P_0 \sin \Omega t \) is

\[
W = \rho A \sum_{n=1}^{N} \alpha_n \int_0^{t_0} \dot{q}_n(\tau)Q(\tau) d\tau
\]  

(11.4.49)

and the energy \( W_d \) dissipated by material damping is

\[
W_d = 2\rho A \sum_{n=1}^{N} \alpha_n \zeta_n \omega_n \int_0^{t_0} \dot{q}_n^2(\tau) d\tau
\]  

(11.4.50)

where, for our numerical simulations, \( N = 10 \) (the first ten normal modes), and \( t_0 = 1 \text{ second} \) (typical
welding time). Figure 11.27 plots $W_d$ as a function of overhang length $L$ of the bus-bar for $h = 0.9$ mm with four different modal damping ratios. Note that $W > W_d$, however their differences are so small that they basically overlap each other on the plots, indicating that the work required to set the overhang into longitudinal motion is negligible. Key findings of the simulation results are summarized as follows:

- The minimum length for which the first longitudinal natural frequency ($\omega_1$) of the overhang matches with the ultrasonic excitation frequency ($\Omega = 20$ kHz) is 43.88 mm. In this case, there can be a large amount of vibrational energy loss. However, since $L \leq 10$ mm in current bus-bar designs, this resonance will not occur in practice.

- For $L \leq 20$ mm, $W_d$ due to longitudinal vibration of the overhang is less than 10 J within the range of modal damping ratio values. Note that $\omega_1 = 43.8$ kHz for $L = 20$ mm.

- In practice, for $L \leq 10$ mm, $W_d$ due to longitudinal vibration of the overhang is negligibly small (less than 1 J) for the range of modal damping ratio values considered.

- In general, $W_d$ increases with increasing damping. However, $W_d$ decreases with increasing damping near resonance, as discussed in Chapter 10 for the single degree of freedom system.

$$W_d = \int_0^{t_0} F \cdot \ddot{q} \, dt$$

Figure 11.27 Energy ($W_d$) dissipated by material damping during longitudinal vibration of the overhang for $h = 0.9$ mm with different modal damping ratios.

**Energy Loss Due to Flexural Vibration**

Due to undesirable vibration of the anvil in the direction normal to the welding surface, the overhang of the bus-bar is subjected to transverse excitation at the welding frequency$^6$. The flexural vibration of the overhang caused by this excitation is suspected to lower the weld strength of battery tabs by channeling a significant amount of energy to the vibrational energy of the overhang. In this section, the amount of energy dissipated by material damping due to the flexural vibration of the overhang is estimated. From Section 11.4.2, the mechanical work $W$ performed by the distributed inertial body force $F(x, t) = F_0 \sin \Omega t$ during the flexural vibration of the overhang is

$$W = \rho A \sum_{n=1}^{N} \alpha_n \int_0^{t_0} \ddot{q}_n(\tau)Q_n(\tau) \, d\tau$$

$^6$ Sub- and super harmonics of 20 kHz are occasionally observed in experiments, however their spectral power is much lower.
and the energy $W_d$ dissipated by material damping is

$$W = 2\rho A \sum_{n=1}^{N} \alpha_n \zeta_n \omega_n \int_{0}^{t_0} \dot{q}_n^2(t) \, dt$$

(11.4.52)

where $N = 10$ and $t_0 = 1$ second are used in the numerical simulations.

Shown in Figures 11.28 and 11.29 are the plots of $W_d$ for the flexural vibration of the overhang as a function of its length $L$ for four different modal damping ratios and different bus-bar thickness (from 0.4 to 0.9 mm). Again, note that $W > W_d$, but their differences are very small. Every peak in the plots corresponds to a length $L$ for which the coupon overhang will resonate at the welding frequency (20 kHz). It should be noted that simulation results presented in the work are based on linear elastic vibration theory from which resonance merely predicts the conditions under which significant energy loss could occur.

- Unlike the case of longitudinal vibration of the overhang (see Figure 11.27) in which the first resonant peak occurs when $L$ is large (> 40 mm) which does not happen in practice, resonance of flexural vibration can occur when $L$ is between 4 and 6 mm for the range of coupon thickness considered.
- Significant vibrational energy loss (of the order of hundreds of joules) can occur when one of the flexural natural frequencies of the overhang is close to the excitation frequency $\Omega = 20$ kHz. For example, with thickness $h = 0.6$ mm, the 1st natural frequency $\omega_1 = 20.225$ kHz for $L = 4.1$ mm, the 2nd natural frequency $\omega_2 = 20.083$ kHz for $L = 10.3$ mm, and the 3rd natural frequency $\omega_3 = 19.607$ kHz for $L = 17.3$ mm. When $h = 0.9$ mm, $\omega_1 = 19.607$ for $L = 5.1$ mm and $\omega_2 = 20.131$ for $L = 12.6$ mm. Such energy loss could account for reduced peel tensile strength.
- Based on the results of our analysis, the overhang of the bus-bar should be kept as short as possible to minimize vibrational energy loss due to the flexural vibration of the overhang during welding. For the coupons under study ($h = 0.9$ mm), an overhang length of 2 mm or smaller results in an insignificant amount of energy loss.

**Estimation of Material Damping in Bus-bar Coupon**

Characteristics of material damping in mechanical systems can be estimated by theory of vibration. One of the most commonly employed techniques is the logarithmic decrement method [19]. Consider the free vibration of a single degree-of-freedom spring-mass-damper system, see Figure 10.3 in Chapter 10. The equation of motion is

$$m\ddot{x} + c\dot{x} + kx = 0$$

(11.4.53)

with initial conditions of $x(0) = x_0$ and $\dot{x}(0) = v_0$. We note that this simple model is fundamentally the same as those modal equations in Eqs. (11.4.18) and (11.4.42) for the longitudinal and flexural vibration of the bus-bar overhang, respectively. The free response of the model can be obtained as

$$x(t) = X e^{-\zeta \omega_n t} \sin(\omega_d t + \phi)$$

(11.4.54)

where, $\omega_n = \sqrt{k/m}$, $\zeta = c/2m\omega_n$ (to be estimated), and $\omega_d = \omega_n\sqrt{1 - \zeta^2}$ have been previously defined, and the amplitude $X$ and phase $\phi$ are obtained by imposing the initial conditions

$$X = \sqrt{\left(\frac{v_0 + \omega_n \zeta x_0}{\omega_d}\right)^2 + \left(\frac{x_0}{\omega_d}\right)^2} \quad \phi = \tan^{-1}\left(\frac{x_0 \omega_d}{v_0 + \omega_n \zeta x_0}\right)$$

(11.4.55)
Figure 11.28  Energy \( (W_d) \) dissipated by material damping during flexural vibration of the overhang for \( h = 0.4, 0.5, \) and \( 0.6 \text{ mm} \) with different modal damping ratios.  See Figure 11.27 for legend.
Figure 11.29 Energy ($W_d$) dissipated by material damping during flexural vibration of the overhang for $h = 0.7$, 0.8, and 0.9 mm with different modal damping ratios. See Figure 11.27 for legend.
Figure 11.30 shows a sample plot of the solution given by Eq. (11.4.54). It is noted that the response of the system decays over time because of the presence of the viscous damping, and from Eq. (11.4.54), is enveloped by the curve $Xe^{-\zeta \omega_n t}$. Define the logarithm decrement as

$$\delta = \ln\left(\frac{x_1}{x_2}\right)$$  \hspace{1cm} (11.4.56)

where $x_1 = x(t)$ and $x_2 = x(t + T_d)$, and $T_d = 2\pi / \omega_d$ is the period of oscillation of the damped system. In other words, $\delta$ is the natural logarithm of the ratio of successive amplitudes of the response over one period. From Eq. (11.4.54), the ratio of the amplitudes can be obtained as

$$\delta = \frac{2\pi \zeta}{\sqrt{1 - \zeta^2}}$$  \hspace{1cm} (11.4.57)

Inverting the above expression gives the damping ratio in terms of the measurable variable $\delta$

$$\zeta = \frac{\delta}{\sqrt{4\pi^2 + \delta^2}}$$  \hspace{1cm} (11.4.58)

Note that one can also obtain the following general relations for the logarithmic decrement

$$\delta = \ln(x_k / x_{k+1}) \hspace{1cm} n\delta = \ln(x_k / x_{k+n}) \hspace{1cm} k = 1, 2, 3, \cdots$$  \hspace{1cm} (11.4.59)

The above relations allow a means of averaging the values of $\delta$ from experimental data. In practice, one would estimate the free vibration amplitudes from the response curve and then average the calculated values of $\delta$. It should be noted that the application of the logarithmic decrement formulas Eqs. (11.4.56) and (11.4.59) is feasible only if the peaks in the response curve can be clearly and easily identified. The logarithmic decrement method can also be applied to the velocity data (laser vibrometer in the experiments collect velocity data). In other words, one can similarly define

$$\delta = \ln(v_1 / v_2)$$  \hspace{1cm} (11.4.60)

and apply the same approach to determine the damping ratio as outlined above.
Experiments were set up to estimate the material damping characteristics of the bus-bar coupons. Response of the transverse vibration of the coupon was recorded near its bottom middle location by a laser vibrometer, (laser head model: OFV-505). According to the logarithmic decrement method, the damping ratio can be estimated by examining the free response of the coupon. The free response refers to the vibration of the coupon immediately after the welding process is completed, during which residual welding energy in the coupon will be dissipated through material internal damping.

Figure 11.31(a) plots the velocity data of the flexural vibration of the bus-bar coupon obtained from one experiment. Figure 11.31(b) shows the free response of the coupon immediately after the welding process is completed. FFT of this decaying response shows that the fundamental period of oscillation is about 20 kHz with a super-harmonic at about 40 kHz, see Figure 11.32. The super-harmonic results in a modulated signal as seen in Figure 11.31(b), with the decaying response $Xe^{-\zeta \omega_n t}$ governed by the lower dominant frequency (20 kHz). It is noted that Figure 11.32 is typical of a welding process, indicating that it is generally difficult to identify response peaks to effectively apply formulas, Eq. (11.4.56) or (11.4.59), for accurate numerical estimations of the material damping ratio. In such cases, a computer code should be developed to extract the values of the peaks and then curve fit these experimental data to the exponential model $x = ae^{bt}$ (the enveloped response is $Xe^{-\zeta \omega_n t}$) where the curve fitting parameter $b = -\zeta \omega_n$ with $\omega_n = 20$ kHz, from which the damping ratio $\zeta$ can be estimated.

Figure 11.31 (a) Velocity data of bus-bar coupon flexural vibration during welding; (b) zoomed-in view of the velocity data during the free vibration of the coupon after welding.

In order to do the curve fitting, one must first determine the range of data to be considered and this should be examined for every set of data. The curve fitting procedure discussed above was applied to the
data of Figure 11.31(b) data between \( t = 1.763 \) and 1.77. The curve fitting result gives \( b = -3.869 \times 10^2 \), with a goodness of fit \( R \)-squared value, \( R^2 = 0.993 \) \((R^2 = 1\) for perfect fit\). The damping ratio is thus estimated from \(-3.869 \times 10^2 = -\zeta \times (2\pi \times 20 \times 10^3)\) or \( \zeta = 0.0031 \). This is consistent with the published values for copper (Irvine, 2004). We assume that the modal damping ratios are equal in the numerical simulations.

![Figure 11.32 FFT of the free response of the bus-bar coupon vibration (see Figure (b)).](image)

### 11.4.4 Summary and Conclusions

The vibrational energy loss of the bus-bar associated with the longitudinal and flexural vibrations of the overhang (the upper part of the bus-bar extended from the anvil) is studied in the present work by applying one-dimensional continuous vibration models. The overhang of the bus-bar is modeled as a thin bar under both the longitudinal and flexural excitations from the anvil. Experiments were also performed to obtain the damping characteristics of the coupon in order to provide realistic values of the material damping ratio in the modeling. Our results lay a foundation for a scientific understanding of the bus-bar dynamics during ultrasonic welding and its potential impact on the weld strength, thus providing guidelines for design and welding of battery tabs. Major findings are summarized as follows:

1) The energy loss due to longitudinal vibration of the bus-bar overhang is negligible.
2) A substantial amount of energy loss can occur due to the flexural vibration of the bus-bar overhang during welding when the overhang resonates at the welding frequency (about 20 kHz).
3) Vibrational energy loss through the bus bar can be significantly reduced by (a) suppressing the anvil vibration; and (b) optimizing the overhang length to avoid vibration resonance.
4) The energy loss is nil when there is no overhang.
5) The energy loss could account for the reduction of the weld strength.
Nomenclature

\begin{itemize}
  \item \( A \) Cross-sectional area of overhang \((A = dh)\)
  \item \( a \) Sonotrode amplitude
  \item \( a_l \) Displacement of anvil in the longitudinal direction of overhang
  \item \( a_f \) Displacement of anvil in the transverse direction of overhang
  \item \( c \) Viscous damping coefficient
  \item \( c_0 \) Phase velocity \((= \sqrt{E/\rho})\)
  \item \( d \) Width of overhang
  \item \( E \) Young’s modulus (110 GPa for copper)
  \item \( h \) Thickness of overhang
  \item \( I \) Second area moment of inertia of overhang \((I = dh^3/12)\)
  \item \( F_0 \) Inertial body force in the transverse direction of overhang \([\text{kN/m}]\) (see Figure 11.25)
  \item \( P_0 \) Inertial body force in the longitudinal direction of overhang \([\text{kN/m}]\) (see Figure 11.24)
  \item \( L \) Overhang length (see Figure 11.26)
  \item \( u \) Longitudinal displacement of overhang
  \item \( U_n \) Longitudinal normal mode function (see Eq. (11.4.12))
  \item \( w \) Transverse displacement of overhang
  \item \( W \) Work done by excitation force
  \item \( W_d \) Energy dissipation due to material damping
  \item \( W_n \) Flexural normal mode function (see Eq. (11.4.37))
  \item \( x \) Longitudinal coordinate (see Figures 11.24 and 11.25)
  \item \( \gamma_n \) Wavenumber
  \item \( \rho \) Mass density (8,940 kg/m\(^3\) for copper)
  \item \( \Omega \) Sonotrode frequency \([\text{rad/s}]\) \((\Omega = 2\pi f, f = \text{sonotrode frequency [Hz]})\)
  \item \( \omega_n \) Natural frequency of the \( n \)-th mode \([\text{rad/s}]\) (see Eq. (11.4.11) and Eq. (11.4.36))
  \item \( \zeta_n \) Damping ratio for the \( n \)-th mode (see Eq. (11.4.19) and Eq.(11.4.43))
\end{itemize}

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References


